A framework for fracture modeling using implicitly defined enrichments over C^k partitions of unity simultaneously based on finite elements and meshfree nodes

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Presentation topics

- Computation of crack parameters

Some features related to accuracy

- Why look foward continuous partition of unity?

Metodologias que podem ser adequadas para a modelagem de descontinuidades

- Purpose

an innovative approach

Some issues of concern in crack modeling

- Integration of singular functions

- Accuracy in computation of crack parameters

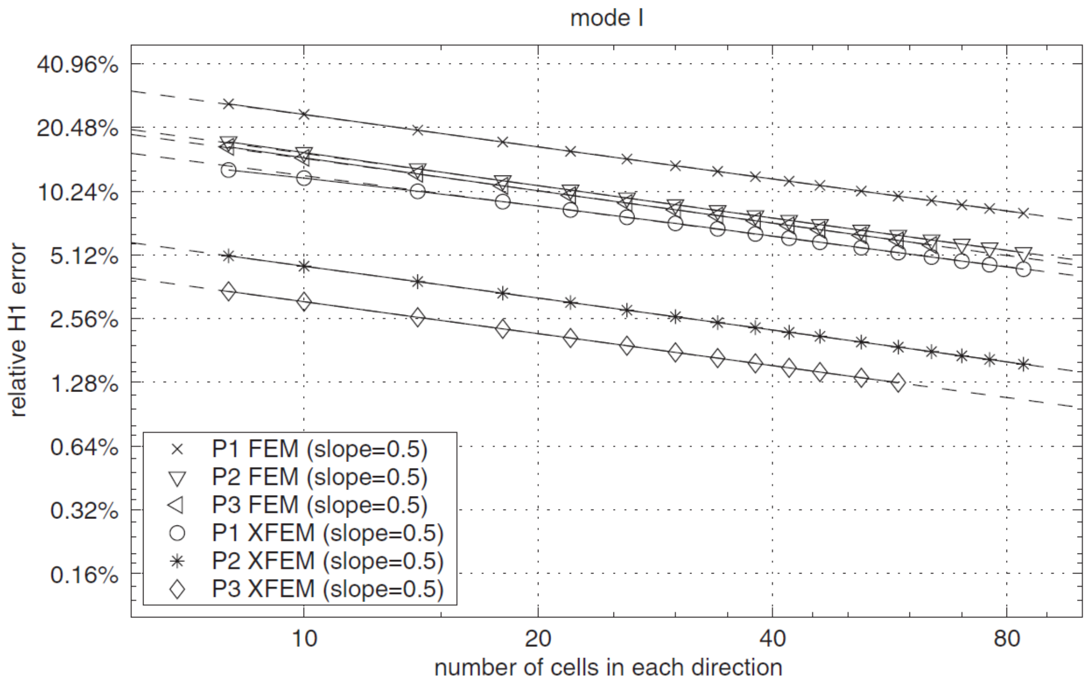
- Flexibility

- Rate of convergence

- Way to performe singular enrichment

- etc.

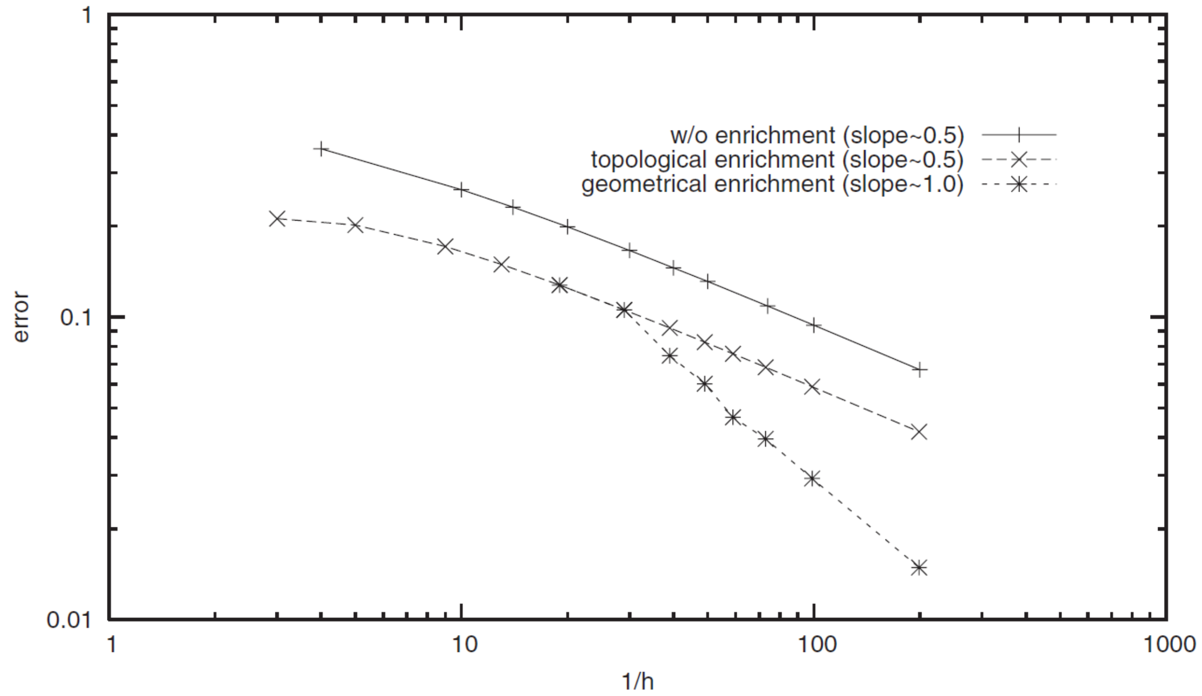
Rate of convergence



Laborde, Pommier, Renard and Salaün, *High-order extended finite element method for cracked domains.* International Journal for Numerical Methods in Engineering, 64 (2005)

Way to perform singular enrichment

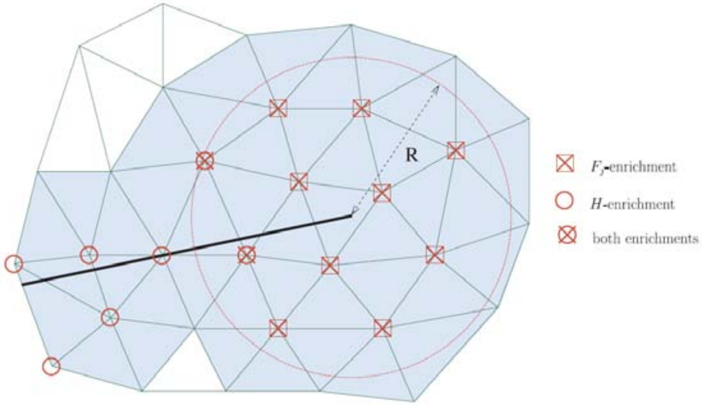
Sabemos que o enriquecimento singular não pode estar atrelado aos nós e temos o problemas de blending

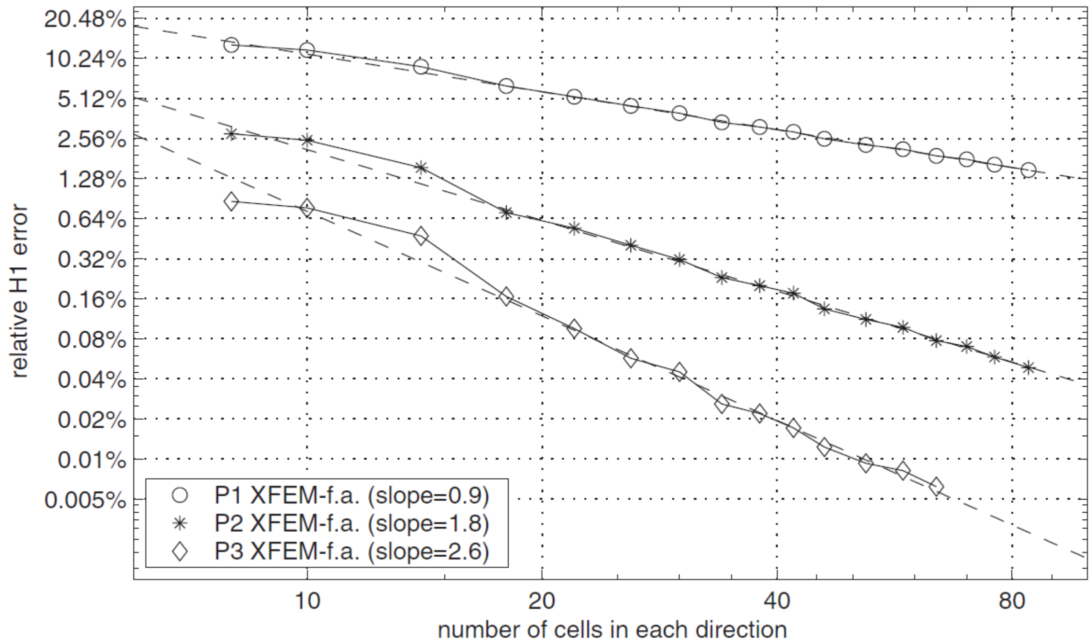


Béchet, Minnebo, Moës and Burgardt, *Improved implementation and robustness study of the X-FEM for stress analysis around cracks.* International Journal for Numerical Methods in Engineering, 64 (2005)

Rate of convergence

Way to perform singular enrichment

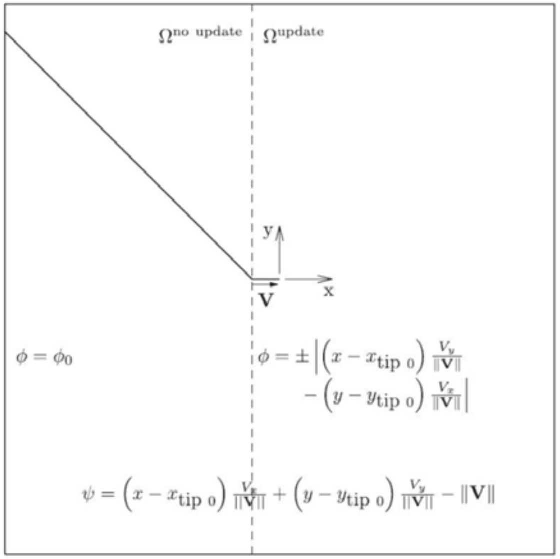




Laborde, Pommier, Renard and Salaün, *High-order extended finite element method for cracked domains.* International Journal for Numerical Methods in Engineering, 64 (2005)

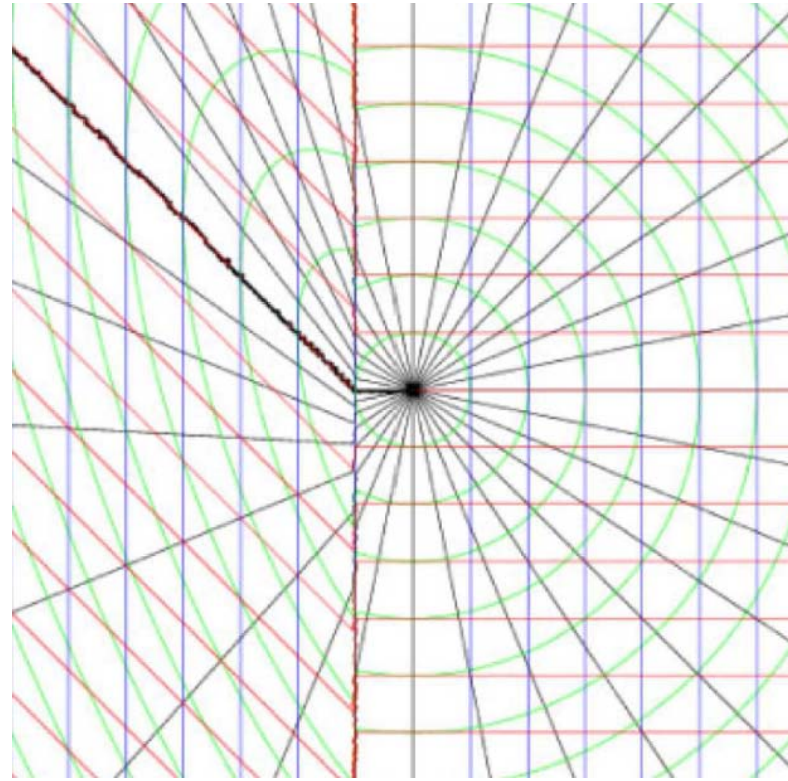
Taracón, Vercher, Giner and Fuenmayor, Enhanced blending elements for XFEM applied to linear fracture mechanics. International Journal for Numerical Methods in Engineering, 77 (2009)

Tracking of crack topology wity level sets



$$r = \sqrt{\phi^2 + \psi^2}$$

$$\theta = \arctan\left(\frac{\phi}{\psi}\right)$$



Ψ : blue r : green

ϕ : red θ : black

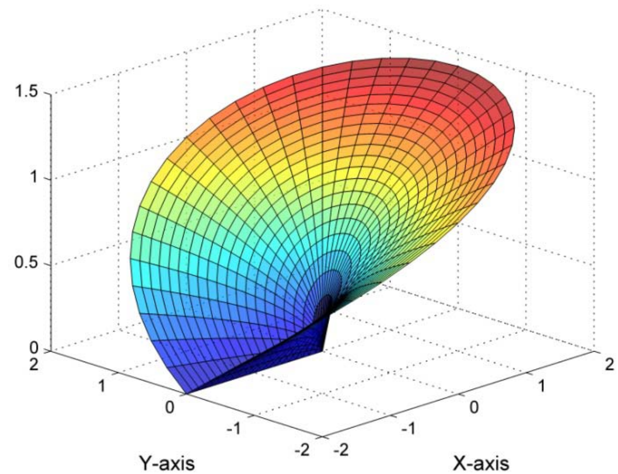
Duflot, *A study of the representation of cracks with level sets.* International Journal for Numerical Methods in Engineering, 70 (2007)

Stolarska, Chopp, Moës and Belytschko, Modelling crack growth by level sets in the extended finite element method. International Journal for Numerical Methods in Engineering, 51 (2001)

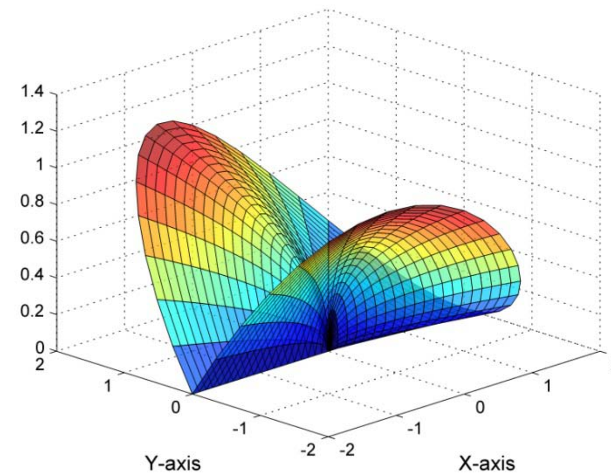
Rate of convergence

Way to perform singular enrichment

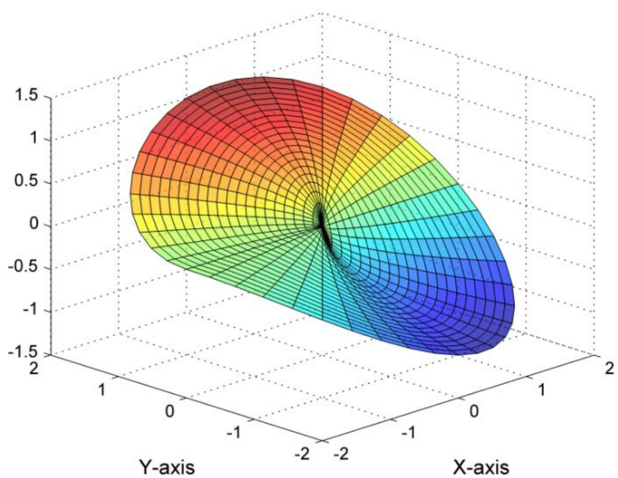
Tracking of crack topology with level sets



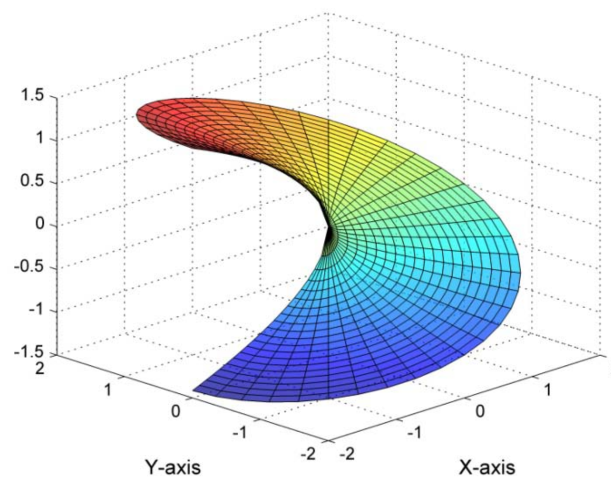
$$\mathcal{L}_1(r,\theta) = \sqrt{r}\,\cos\left(\frac{\theta}{2}\right)$$



$$\mathcal{L}_2(r,\theta) = \sqrt{r}\,\sin\left(\frac{\theta}{2}\right)\sin(\theta)$$



$$\mathcal{L}_3(r,\theta) = \sqrt{r}\,\cos\left(\frac{\theta}{2}\right)\sin(\theta)$$

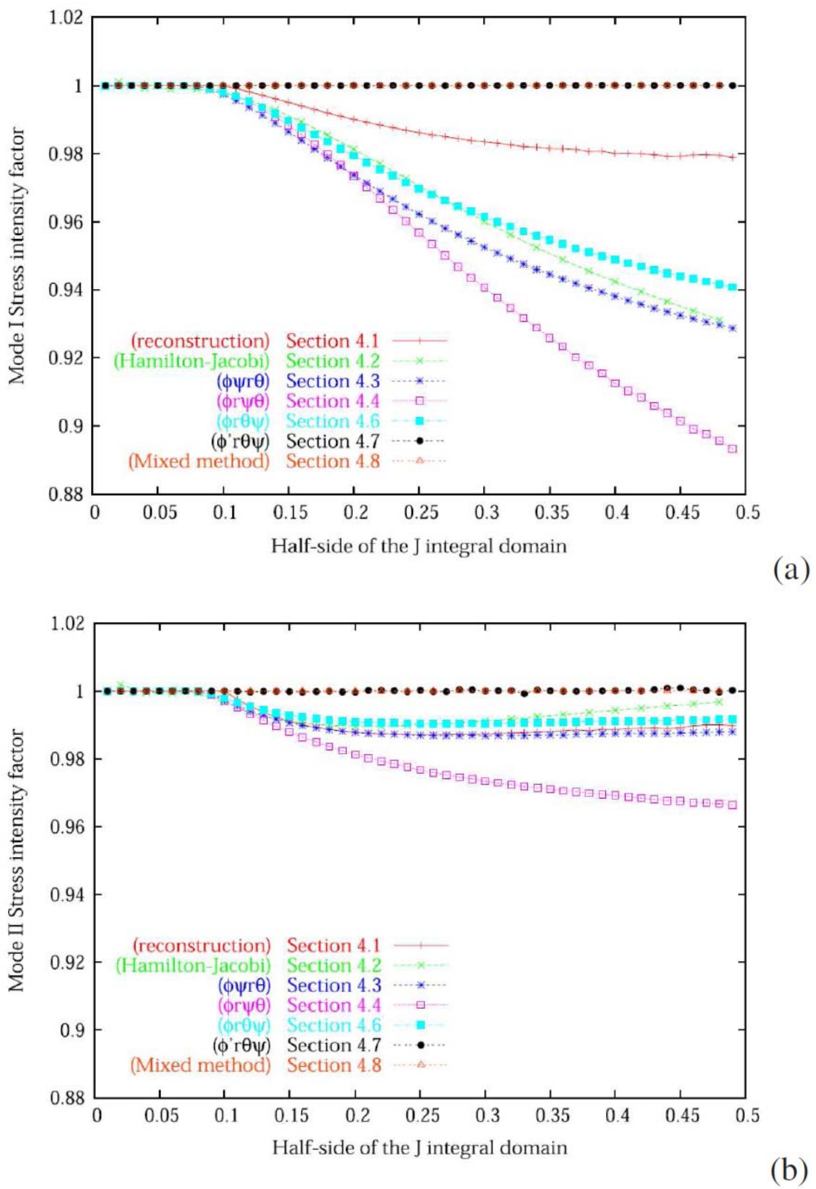


$$\mathcal{L}_4(r,\theta) = \sqrt{r}\,\sin\left(\frac{\theta}{2}\right)$$

Rate of convergence

Way to perform singular enrichment

Tracking of crack topology with level sets

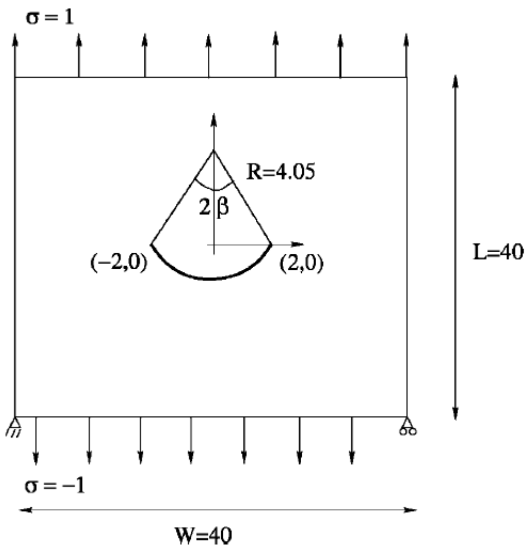


Duflot, *A study of the representation of cracks with level sets.* International Journal for Numerical Methods in Engineering, 70 (2007)

Crack topology approximation

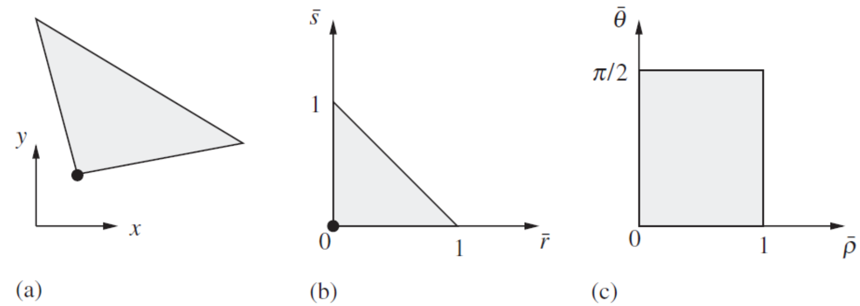
$$\mathbf{u}^{\mathrm{h}}(\mathbf{x}) = \sum_{I \in \mathcal{N}} N_I(\mathbf{x})\mathbf{u}_I + \sum_{I \in \mathcal{N}^{\mathrm{cr}}} \tilde{N}_I(\mathbf{x})(H(f^{\mathrm{h}}(\mathbf{x})) - H(f_I))\mathbf{a}_I + \sum_{I \in \mathcal{N}^{\mathrm{TIP}}} \tilde{N}_I(\mathbf{x}) \sum_{k=1}^{4} (F^{\mathrm{k}}(r,\theta) - F^{\mathrm{k}}(x_I))\mathbf{b}_I^{\mathrm{k}}$$

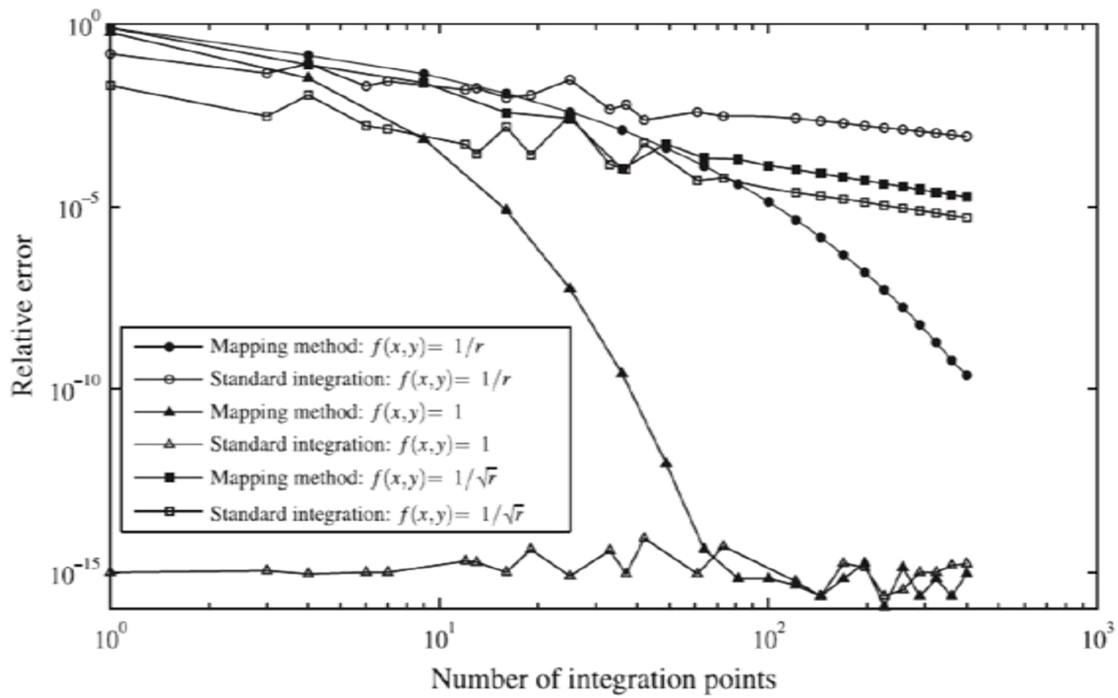
$$f(\xi) = \sum_{I=1}^{6} f_I N_I(\xi)$$



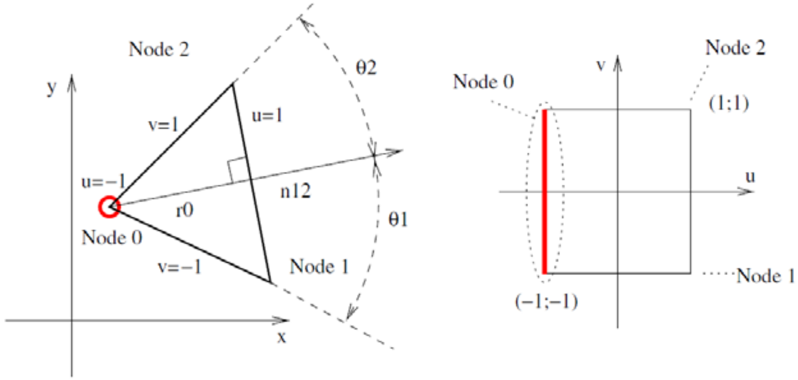
Stazi, Budyn, Chessa and Belytschko, *An extended finite element method with higher-order elements for curved cracks.* Computational Mechanics, 31 (2003)

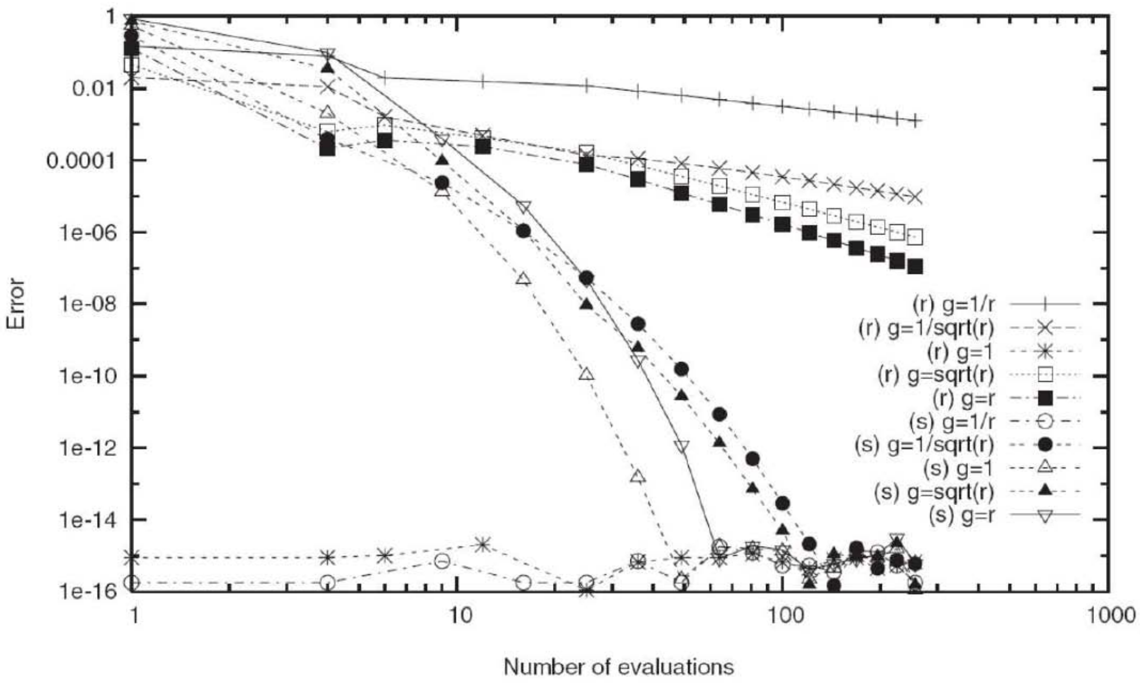
Integration of singular functions





Park, Pereira, Duarte and Paulino, *Integration of singular enrichment functions in the generalized / extended finite element method for three-dimensional problems.* International Journal for Numerical Methods in Engineering, 78 (2009)





Béchet, Minnebo, Moës and Burgardt, *Improved implementation and robustness study of the X-FEM for stress analysis around cracks.* International Journal for Numerical Methods in Engineering, 64 (2005)

Some previous conclusions

Singularity of the weighted moment matrix (Liu 2003)

A good method of shape functions construction shoul satisfy the following basic requirements:

-Arbitrary nodal distribution

-Stability of the algorithm

-Consistency condition

-Compact support

-Efficiency

-Delta function property

-Compatibility

-(Liu, 2003) Mesh free methods: moving beyond the finite element method.

The condition number of the final system matrix scales with the same order over the element size as the standard FEM for arbitrary enrichments. In standard XFEM, hoever, the condition number may significantly increase with refinement, which depends on the particular enrichment.

Partially enriched elements show a systematical error, which hinders an optimal convergence for the standard XFEM.

The evaluation of the MLS functions involves increased amount of computational work.

Overlapping subdomains in order to enable individual enrichments in different parts of the domain, according to the locally present characteristics of the solution.

(Fries and Belytschko 2006, The intrinsic XFEM..., pág1360,

Some alternatives we have

Implicity representation of geometry

Rvachev (1963) wanted to devise a methodology for solving what he termed the inverse problems of analytic geometry: constructing equations and inequalities for given geometric objects while direct problems which mean investigating geometric objects by algebraic equations and inequalities. R-functions operate on real-valued inequalities as differentiable logic operations.

The theory of Rvachev provides means for systematically constructing smooth approximations to distance functions for any closed semi-analytic set and such generalized functions may be applied to interpolation of scattered data.

All closed semi-analytic point sets may be implicitly represented by real valued functions with guaranteed differential properties using the theory of R-functions

Implicit representation of a point set by the zeros fo some real valued functions is not constrained by the topology of the represented set.

Rvachev seminal work

Rvachev (1963) On analytical description of some geometric objects. Reports of academy of sciences.

Rvachev, Sheiko, Shapiro and Tsukanov (2001) Transfinite interpolation over implicitly defined sets. Computer Aided Design

Shapiro (2007) Semi-analytic geometry with R-functions. Acta Numerica

$$f_1 \wedge_\alpha f_2 \equiv \frac{1}{1+\alpha}\left(f_1 + f_2 - \sqrt{f_1^2 + f_2^2 - 2\alpha f_1 f_2}\right)$$

$$f_1 \vee_\alpha f_2 \equiv \frac{1}{1+\alpha}\left(f_1 + f_2 + \sqrt{f_1^2 + f_2^2 - 2\alpha f_1 f_2}\right)$$

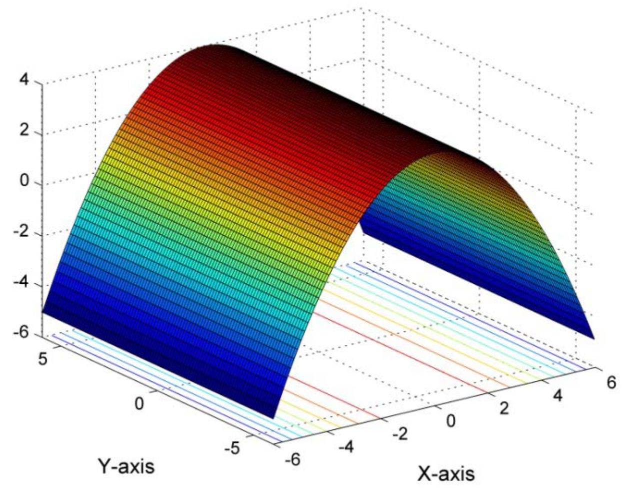
$$-1 \leq \alpha(f_1, f_2) \leq 1$$

$$f_1 \wedge_\alpha^m f_2 \equiv (f_1 \wedge_\alpha f_2)\left(f_1^2 + f_2^2\right)^{m/2}$$

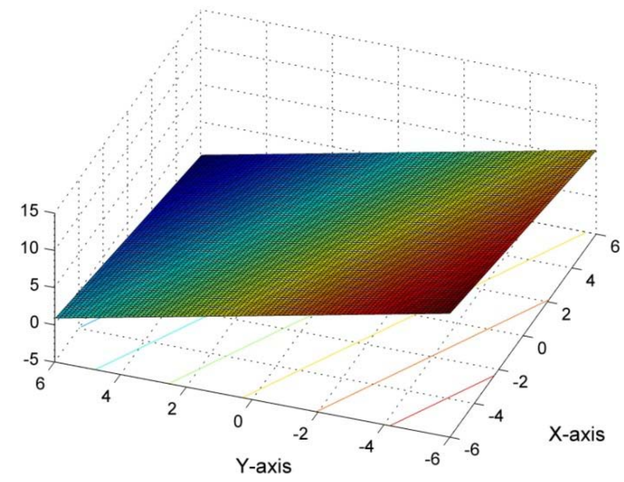
$$f_1 \vee_\alpha^m f_2 \equiv (f_1 \vee_\alpha f_2)\left(f_1^2 + f_2^2\right)^{m/2}$$

Rvachev, On the analytical description of some geometric objects. Reports of Ukrainian Academy of Sciences, 153 (1963)

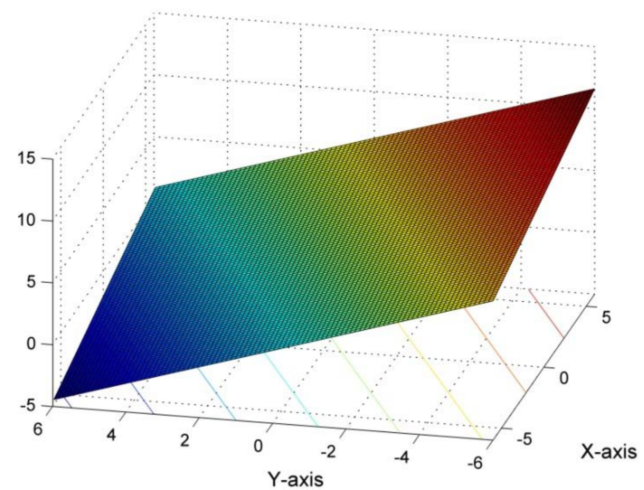
Implicity representation of geometry



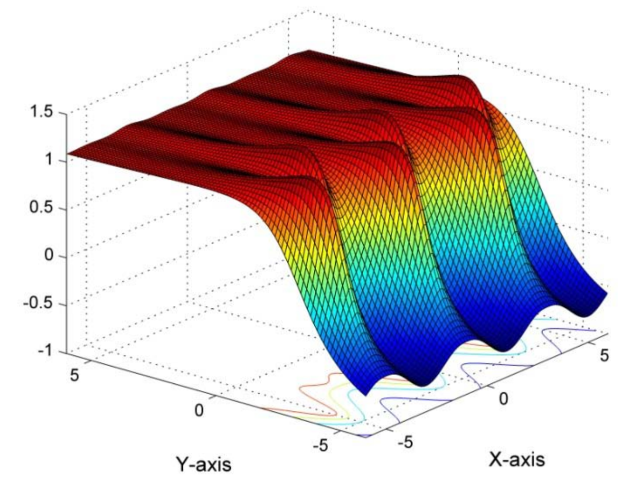
$$f_1(x,y) = \frac{16-x^2}{4}$$



$$f_2(x,y) = \frac{(y-3)(x_a-2)-(x-2)(y_a-3)}{\sqrt{(x_a-2)^2+(y_a-3)^2}}$$

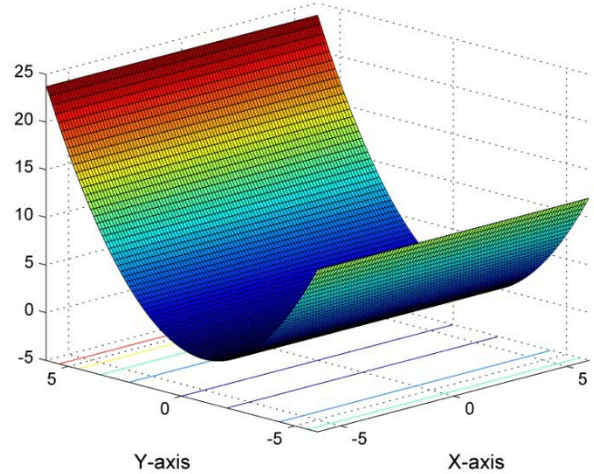


$$f_3(x,y) = -\frac{(y-y_a)(x_a+2)+(x-x_a)(3-y_a)}{\sqrt{(x_a+2)^2+(3-y_a)^2}}$$

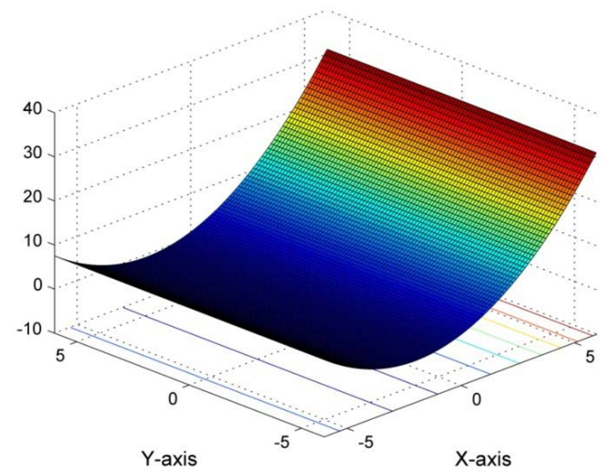


$$f_4(x,y) = \frac{3+y-\sin\left(\frac{\pi x}{2}\right)}{\sqrt{1+\frac{\pi^2}{4}\cos^2\left(\frac{\pi x}{2}\right)+\left(3+y-\sin\left(\frac{\pi x}{x}\right)\right)^2}}$$

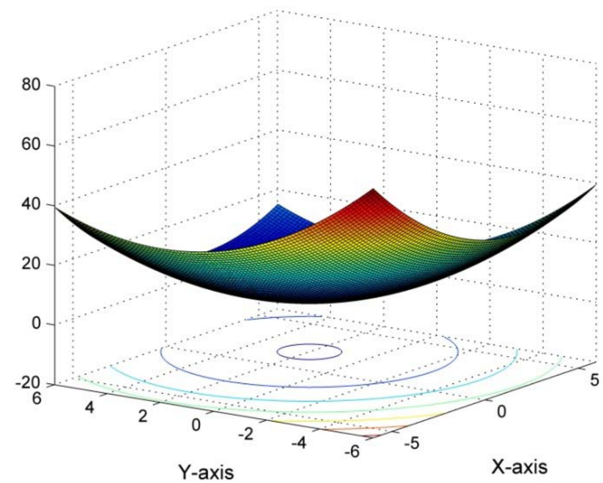
Implicity representation of geometry



$$f_5(x,y) = -\frac{\left(\frac{h}{2}\right)^2 - (y-y_b)^2}{h}$$

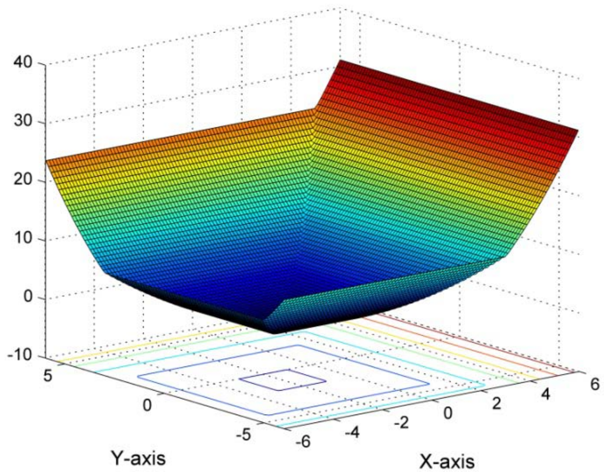


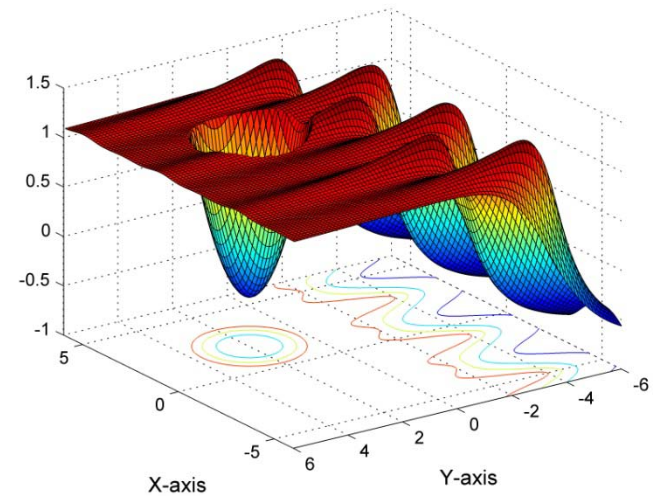
$$f_6(x,y) = -\frac{\left(\frac{l}{2}\right)^2 - (x-x_b)^2}{l}$$



$$f_7(x,y) = \frac{1}{2r}\left((x-x_c)^2 + (y-y_c)^2 - r^2\right)$$

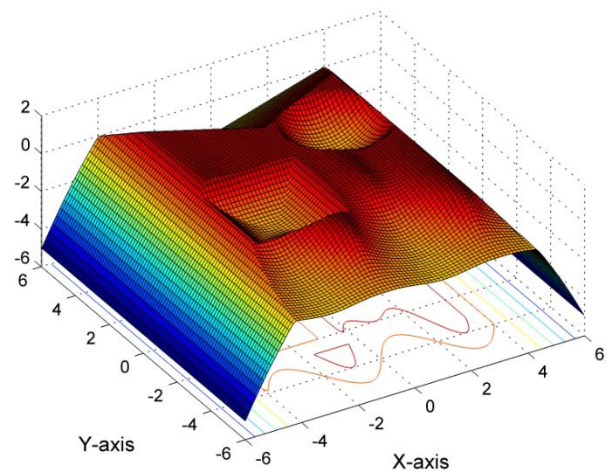
Implicity representation of geometry

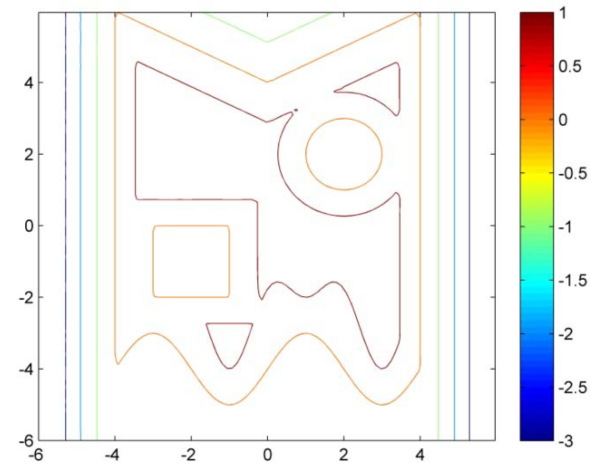




$f_5 \vee_0 f_6$

$f_4 \wedge_0 f_7$

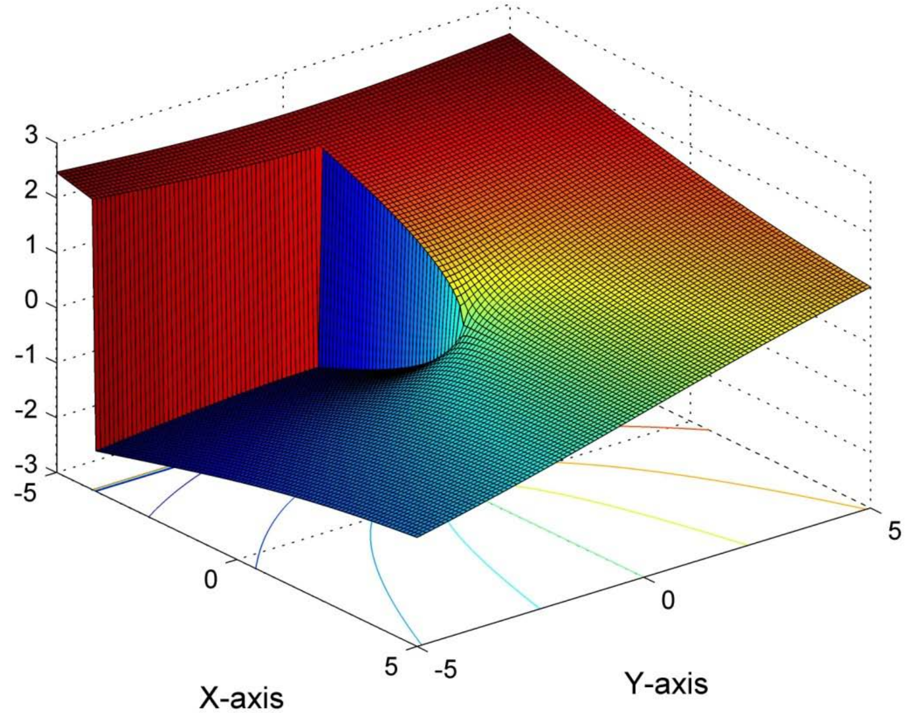


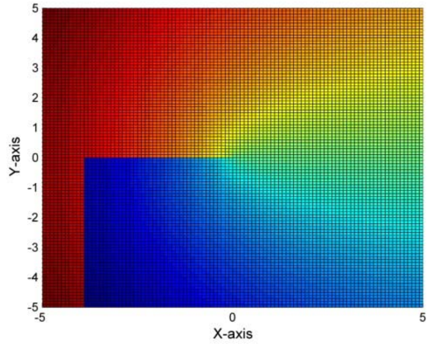


$$\Omega = f_1 \wedge_0 (f_5 \vee_0 f_6) \wedge_0 (f_2 \vee_0 f_3) \wedge_0 (f_4 \wedge_0 f_7)$$

Implicity representation of geometry

Enrichments building with R-functions





Tipos de partição da unidade: características, vantagens e disvantagens

Vantagens da minha abordagem com relação a outras partições da unidade mesh-free: aqui a malha serve para definir conectividade e integração, somente!

Ck-gfem: principais vantagens e desvantagens

O que me levou a optar por isso?

Flexibilidade na definição da descontinuidade com relação ao posicionamento da nuvem sobreposta e poder mover tal característica, aqui tem também a ver com a questão do tipo de enriquecimento (geométrico, segundo Laborde et al. (2005))

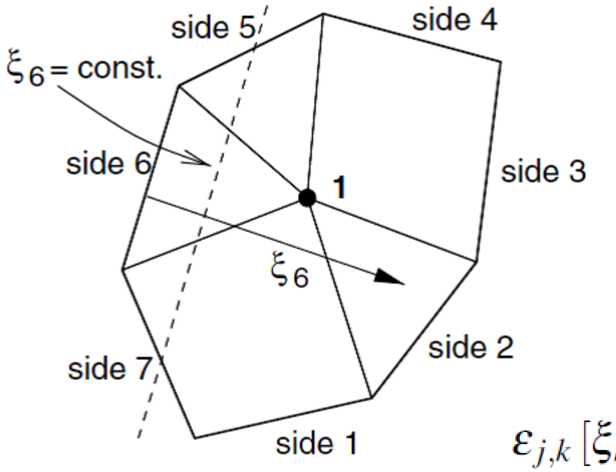
Necessidade de maior regularidade na região da singularidade para evitar erro, procurar estimação de erro em J, por exemplo, e alguma relação com a regularidade dos espaços.

Possibilidade de definir a topologia da descontinuidade usando somente parâmetros geométricos sem necessidade de resolver outro problema de valor no contorno, como level set

Implicity representation of geometry

C^∞ – partiton of unity

C^∞ – partition of unity



$$\xi_k(\mathbf{x}) = \mathbf{n}_{j,k} \cdot (\mathbf{x} - \mathbf{b}_{j,k})$$

$$\varepsilon_{j,k}[\xi_k(\mathbf{x})] = \begin{cases} A e^{-(\xi_k/B)^{-\gamma}} & \text{, if } \quad \xi_k > 0 \\ 0 & \text{, otherwise} \end{cases}$$

$$A = e^{\left(\frac{1-2^\gamma}{\log_e \beta}\right)^{1/\gamma}} \qquad B = h_{j,k}\left(\frac{\log_e \beta}{1-2^\gamma}\right)^{1/\gamma}$$

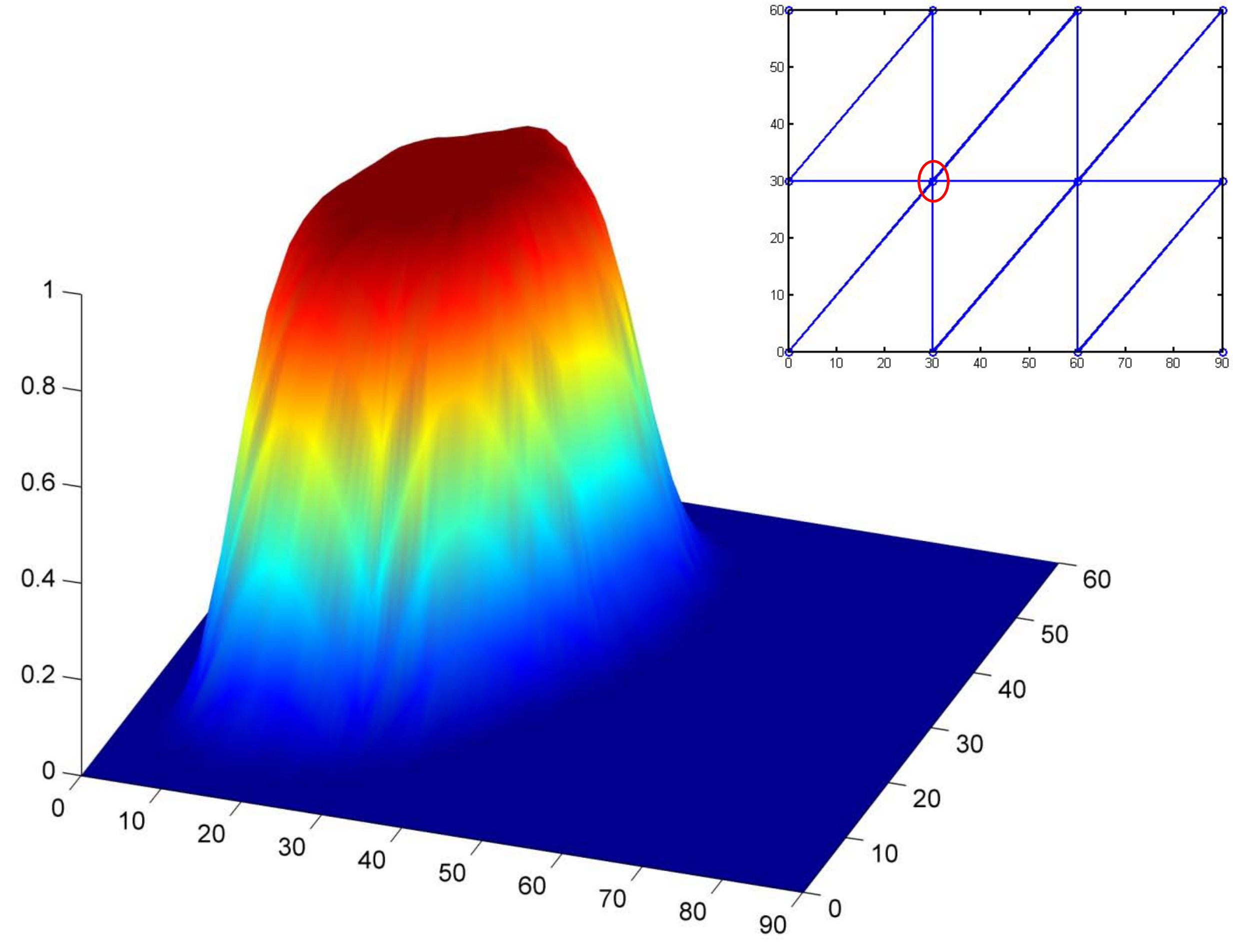
$$\mathcal{W}_j(\mathbf{x}) := \prod_{k=1}^{M_j} \varepsilon_{j,k}(\xi_k) \qquad \mathcal{N}_j(\mathbf{x}) = \frac{\mathcal{W}_j(\mathbf{x})}{\sum_{\beta(\mathbf{x})} \mathcal{W}_\beta(\mathbf{x})} \qquad \beta(\mathbf{x}) \in \{\gamma \mid \mathcal{W}_\gamma(\mathbf{x}) \neq 0\}$$

Edwards, C∞ finite element basis functions, Report 45, Institute for Computational Engineering and Sciences – The University of Texas at Austin, 2006

Duarte, Migliano and Quaresma, Arbitrarily smooth generalized finite element approximations. Computer Methods in Applied Mechanics and Engineering, 196 (2006)

Implicity representation of geometry

C^{∞} – partiton of unity



Xuan, Lassila, Rozza and Quarteroni, *On computing upper and lower bounds on the outputs of linear elasticity problems approximated by the smoothed finite element method.* International Journal for Numerical Methods in Engineering, -- (2004)

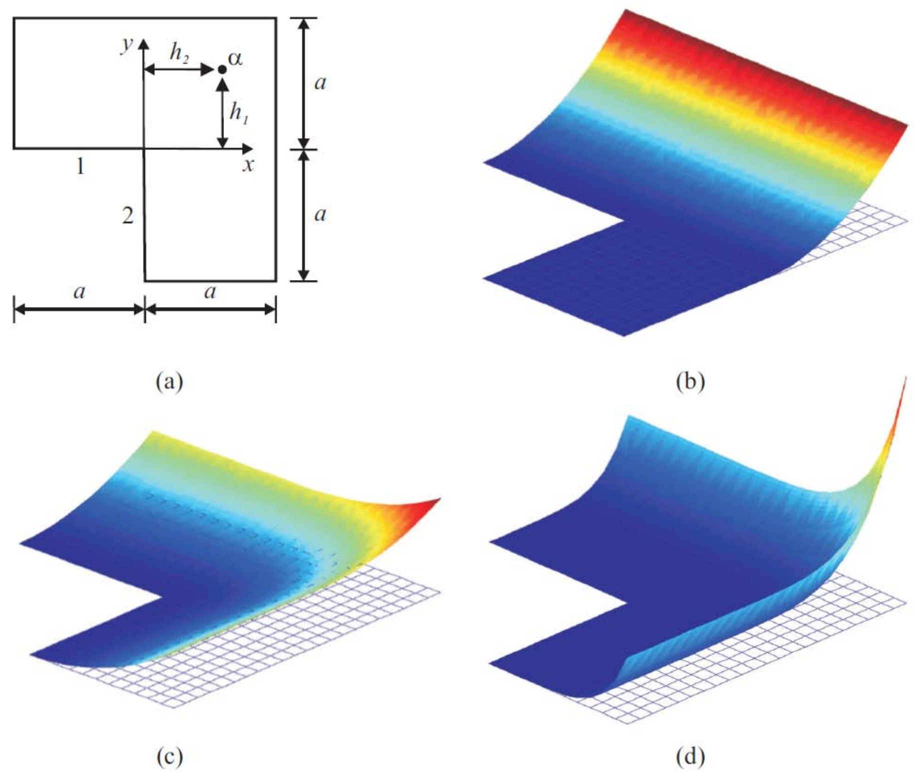
Implicity representation of geometry

C^∞ – partiton of unity

C^k – partiton of unity

C^k – partition of unity

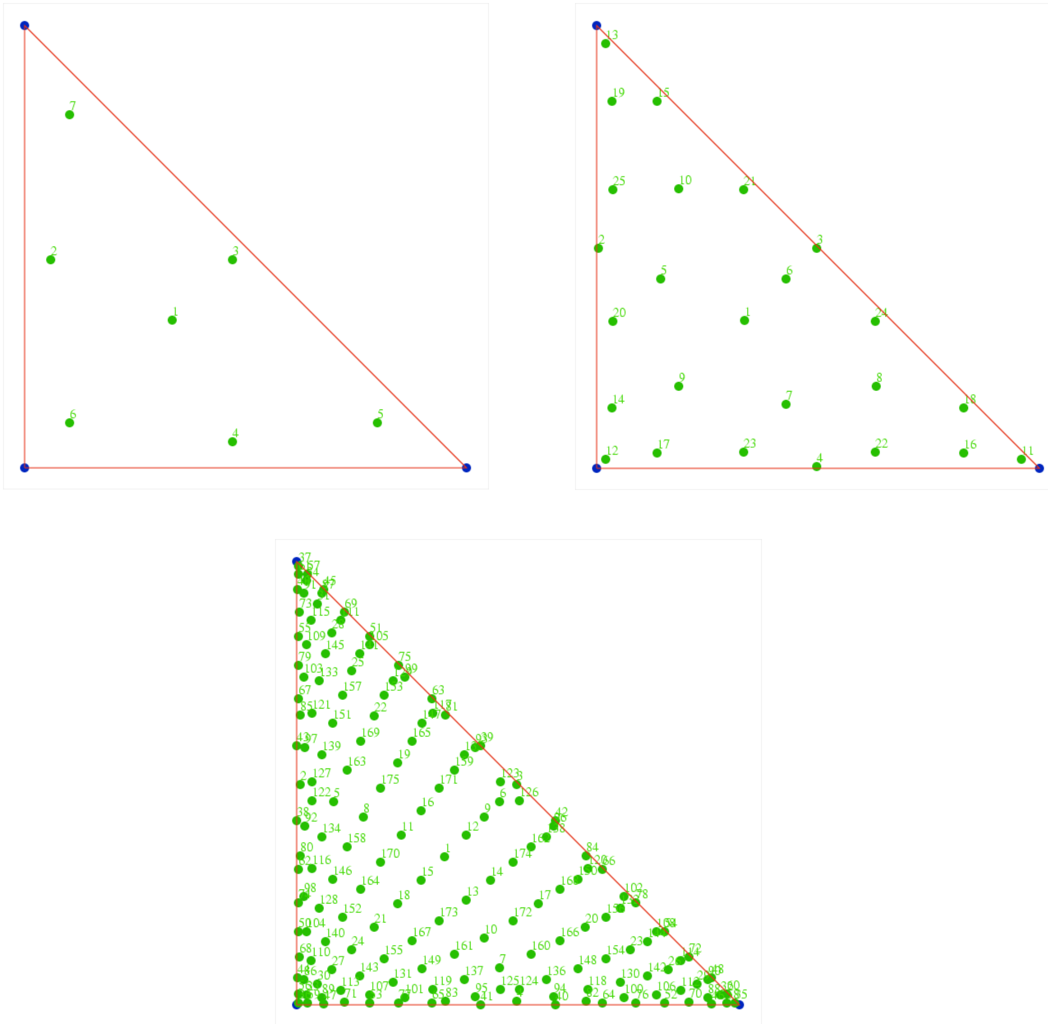
$$\left(f_1 \vee_0^k f_2\right) := \left(f_1 + f_2 + \sqrt{f_1^2 + f_2^2}\right)\left(f_1^2 + f_2^2\right)^{\frac{k}{2}}$$



Duarte, Migliano and Quaresma, Arbitrarily smooth generalized finite element approximations. Computer Methods in Applied Mechanics and Engineering, 196 (2006)

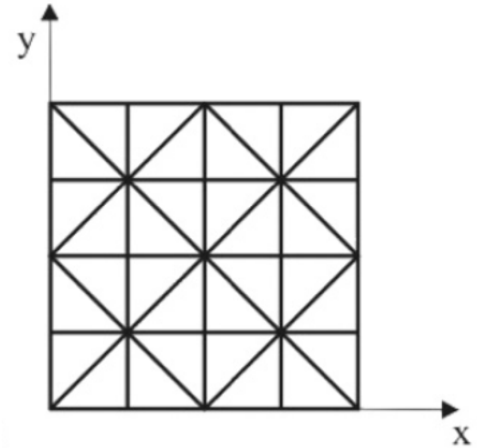
Mendonça, Barcellos and Torres, *Analysis of anisotropic Mindlin plate model by continuous and non-continuous GFEM.* Finite Element in Analysis and Design, 47 (2011)

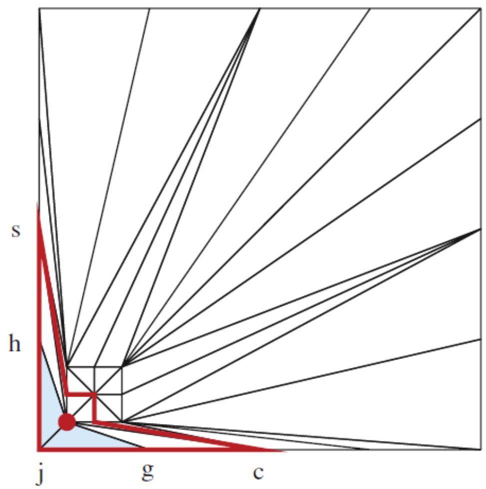
Numerical integration of regular functions



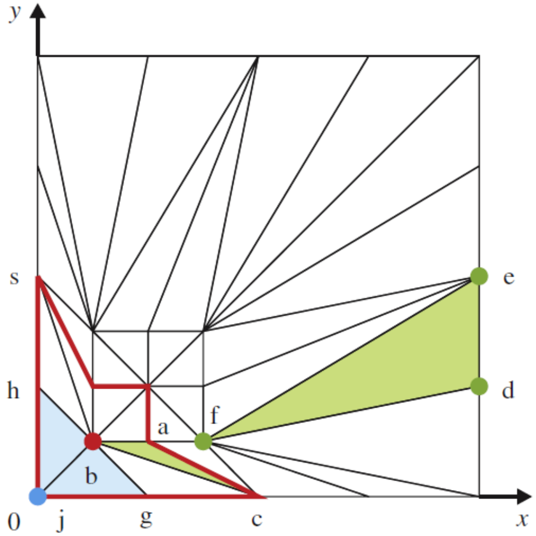
Wandzura and Xiao, *Symmetric quadrature rules on a triangle.* Computer and Mathematics with Applications, 45 (2003)

Peformance against mesh distortion





Mesh 4x4, $s = 0.25$



Mesh 4x4, $s = 0.5$

Mendonça, Barcellos and Torres, *Analysis of anisotropic Mindlin plate model by continuous and non-continuous GFEM.* Finite Element in Analysis and Design, 47 (2011)

Implicity representation of geometry

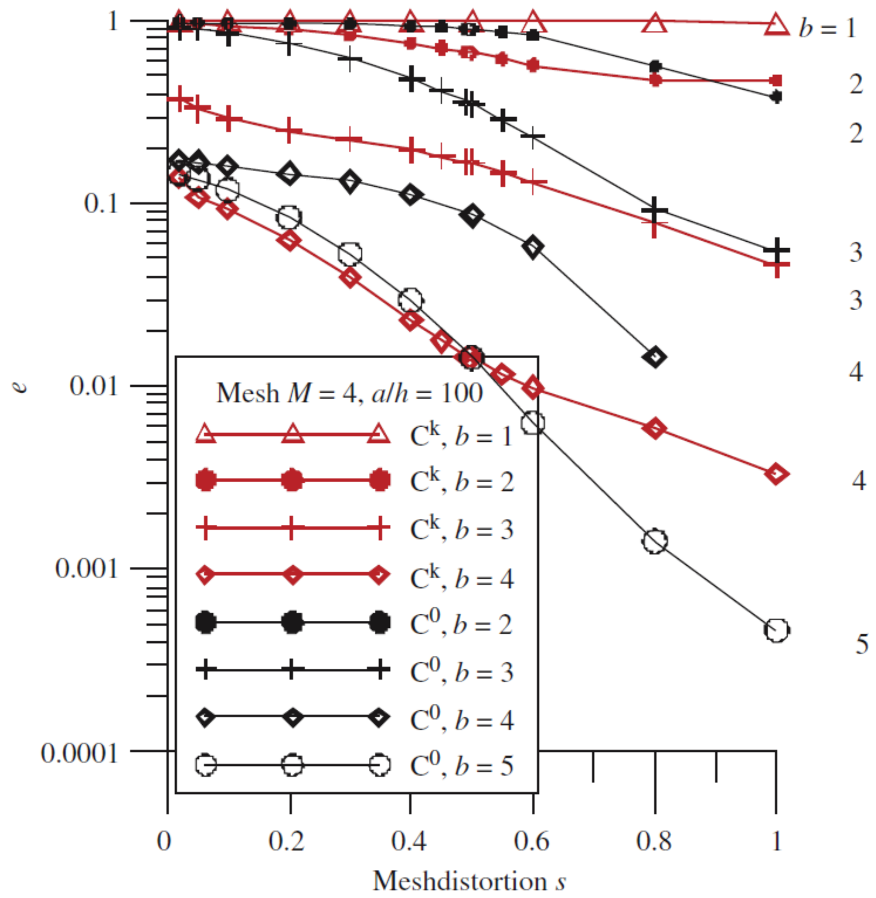
C^∞ – partiton of unity

C^k – partiton of unity

Numerical integration of regular functions

Performance against mesh distortion

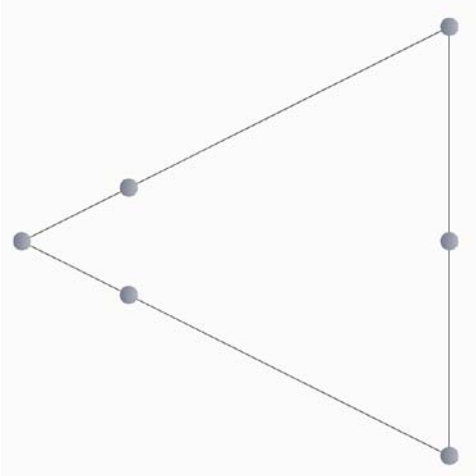
$$e = \sqrt{\frac{E_0 - E}{E_0}}$$

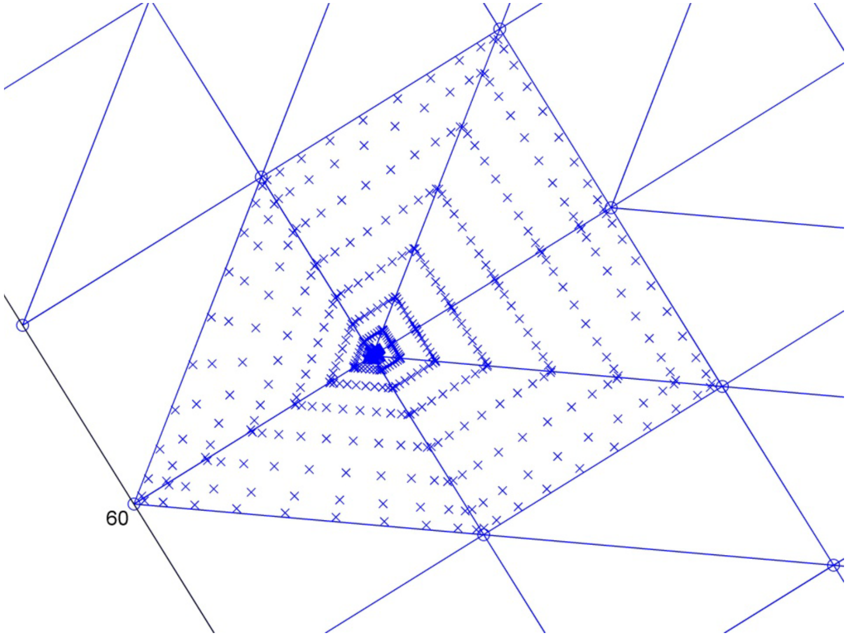


Mendonça, Barcellos and Torres, *Analysis of anisotropic Mindlin plate model by continuous and non-continuous GFEM.* Finite Element in Analysis and Design, 47 (2011)

Numerical integration of singular functions

Procurar aquela definição de fracamente singular no artigo de Park e Duarte





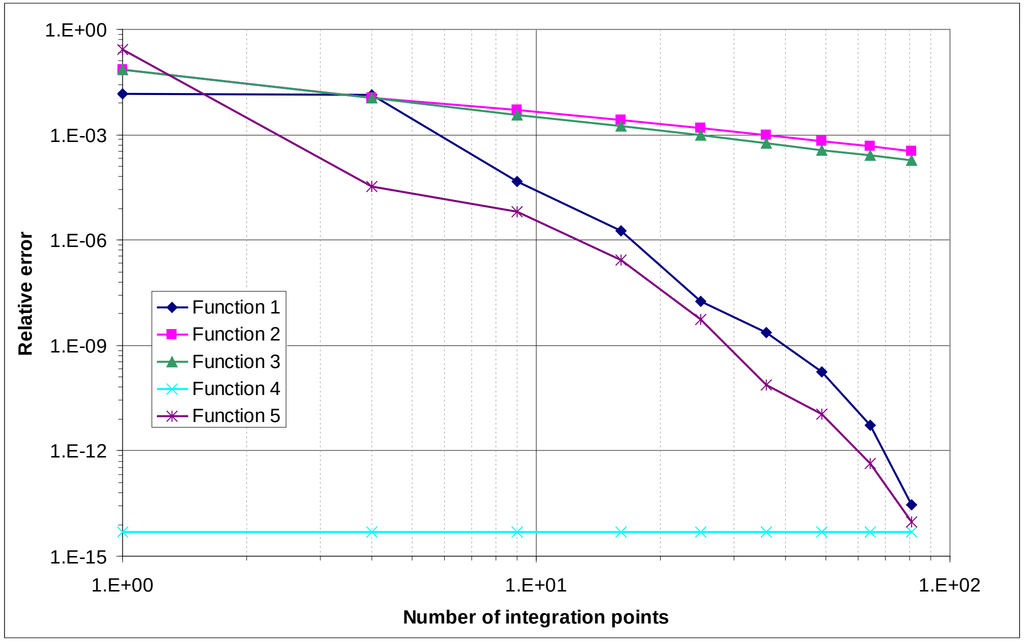
$$\text{function } 1 = \frac{1}{r}$$

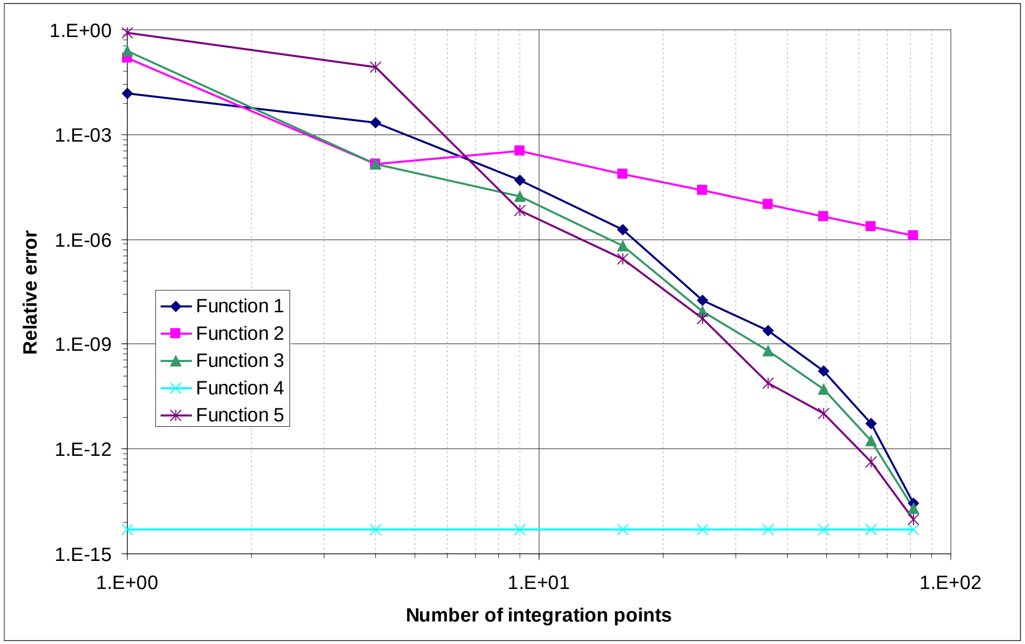
$$\text{function } 2 = \frac{1}{r^{\frac{2}{3}}}$$

$$\text{function } 3 = \frac{1}{\sqrt{r}}$$

$$\text{function } 4 = 1$$

$$\text{function } 5 = r$$



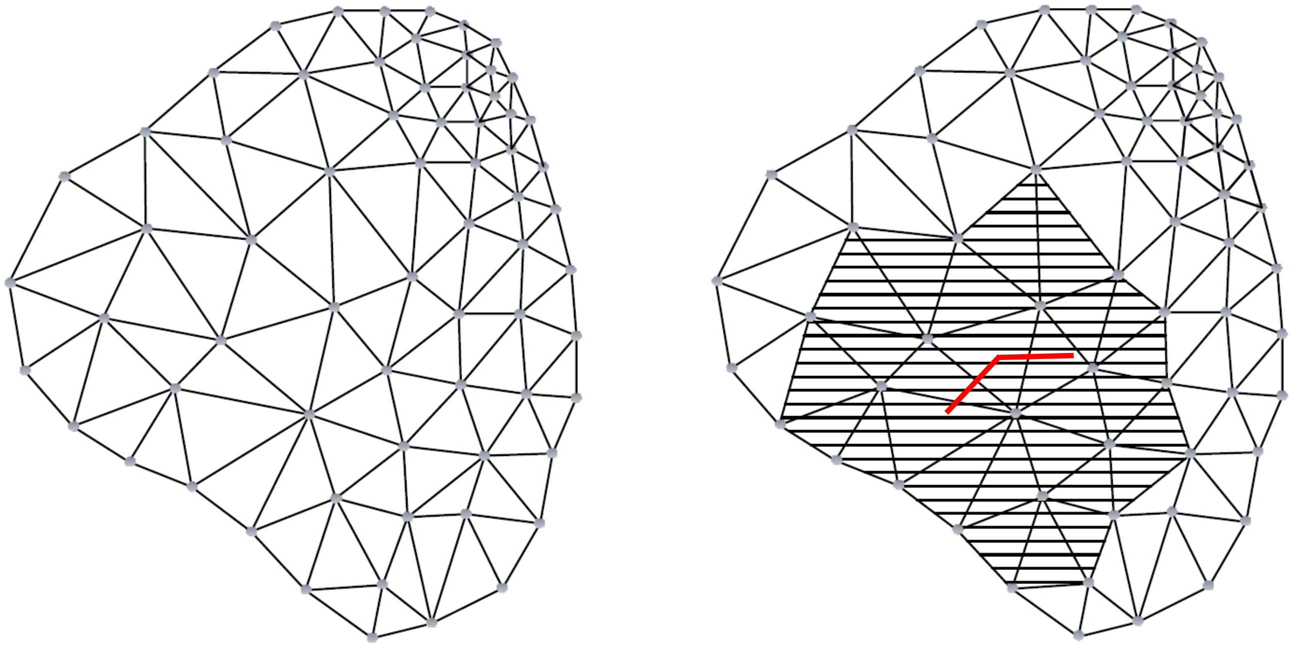


The authors also addressed the merging of cracktip analytical and heaviside step function enrichments.

It transforms domain integrals to equivalente contour integrals.The method is applicable only to elements for which all nodes of the element are enriched and for which all enriched degrees of freedom at all nodes of the element are the same. additionally, the enrichment function must be self-equilibrated.

Boundary integration of Ventura, Gracie and Belytschko (2009)

Ventura, Gracie and Belytschko (2009) Fast integration and weight function blending in the extended finite element method. International Journal for Numerical Methods in Engineering



Implicity representation of geometry

C^∞ – partiton of unity

C^k – partiton of unity

Numerical integration of regular functions

Performance against mesh distortion

Numerical integration of singular functions

Coupling between mesh-based and meshfree PoU

Coupling between mesh-based and meshfree PoU

$$\mathcal{N}_j(\mathbf{x}) = \begin{cases} \dfrac{\mathcal{W}_j^{fe}(\mathbf{x})}{\sum_{\beta\in\mathcal{I}_{fe}(\mathbf{x})}\mathcal{W}_\beta^{fe}(\mathbf{x}) + \sum_{\gamma\in\mathcal{I}_{mf}(\mathbf{x})}\mathcal{W}_\gamma^{mf}(\mathbf{x})}, & \text{if } j\in\mathcal{I}_{fe} \\ \dfrac{\mathcal{W}_j^{mf}(\mathbf{x})}{\sum_{\beta\in\mathcal{I}_{fe}(\mathbf{x})}\mathcal{W}_\beta^{fe}(\mathbf{x}) + \sum_{\gamma\in\mathcal{I}_{mf}(\mathbf{x})}\mathcal{W}_\gamma^{mf}(\mathbf{x})}, & \text{if } j\in\mathcal{I}_{mf} \end{cases}$$

$$\mathcal{I}_{fe}(\mathbf{x}) = \{\beta\in\mathcal{I}_{fe} : \mathcal{W}_\beta(\mathbf{x})\neq 0\}$$
$$\mathcal{I}_{mf}(\mathbf{x}) = \{\beta\in\mathcal{I}_{mf} : \mathcal{W}_\beta(\mathbf{x})\neq 0\}$$

$$\mathcal{I}_{mf,fe} = \mathcal{I}_{fe}\cup\mathcal{I}_{mf}$$

$$\{\mathcal{N}_j\}_{j\in\mathcal{I}_{mf,fe}}$$

$$\sum_{j\in\mathcal{I}_{mf,fe}}\mathcal{N}_j(\mathbf{x}) = 1 \quad \forall\,\mathbf{x}\in\Omega$$

Duarte, Migliano and Baker, A technique to combine meshfree- and finite element-based partition of unity approximations, Structural Research Series 638, Department of Civil and Environmental Engineering – UIUC, 2005

Ventura, Gracie and Belytschko, Fast integration and weight functions blending in the extended finite element method. International Journal for Numerical Methods in Engineering, 77 (2009)

Ventura, Gracie and Belytschko, *An extended finite element method with higher-order elements for curved cracks.* Computational Mechanics, 31 (2009)

Imposição de condições de contorno.

Compatibilidade entre partição contínua e funções de forma de elementos finitos convencionais

integração

Geometria sólida construtiva e geração de funções de aproximação definidas implicitamente e o que isso pode representar como possibilidades.

Espera-se melhorias devido à continuidade das derivadas, questões relacionadas à continuidade da derivada e alguma coisa a se pensar a cerca dos movimentos de corpo rígido.

Falar um pouco sobre estimadores implícitos em nuvens

Falar da proposta de estender o estimador do Felício para funções CK

Mendonça, Barcellos and Torres, *Analysis of anisotropic Mindlin plate model by continuous and non-continuous GFEM.* Finite Element in Analysis and Design, 47 (2011)

Cloud-based implicit residual error estimator

$$\mathscr{B}(\mathbf{e}_p, \mathbf{v}) = \mathscr{R}(\mathbf{v}) = \mathscr{R}\left(\mathbf{v}\sum_{j=1}^{N}\mathcal{N}_j\right) = \sum_{j=1}^{N}\mathscr{R}(\mathcal{N}_j\mathbf{v})$$

$$\mathscr{R}(\mathcal{N}_j\mathbf{v}) = 0 \quad \omega_j \cap supp(v) = \emptyset$$

$$\mathscr{B}_{\omega_j}\left(\mathbf{e}_p^{\omega_j}, \mathbf{v}^{\omega_j}\right) = \mathscr{R}_{\omega_j}(\mathcal{N}_j\mathbf{v}^{\omega_j}) \;\forall\, \mathbf{v}^{\omega^j} \in \mathscr{V}(\omega_j)$$

$$\mathscr{R}_{\omega_J}(\mathcal{N}_j\mathbf{v}^{\omega_j}) = \mathscr{L}_{\omega_j}(\mathcal{N}_j\mathbf{v}^{\omega_j}) - \mathscr{B}_{\omega_j}(\mathbf{u}_p, \mathcal{N}_j\mathbf{v}^{\omega_j})$$

Parés, Díez and Huerta, *Subdomain-based flux-free a posteriori error estimators.* Computer Methods in Applied Mechanics and Engineering, 195 (2006)

Barros, Barcellos and Duarte, *Subdomain-based flux-free a posteriori estimator for generalized finite element method.* Proceedings of the thirth iberian-latin-american congress on computational methods in engineering – XXX CILAMCE (2009)

Barros, Proença and Barcellos, *On error estimator and p-adaptivity in the generalized finite element method.* International Journal for Numerical Methods in Engineering, 60 (2004)

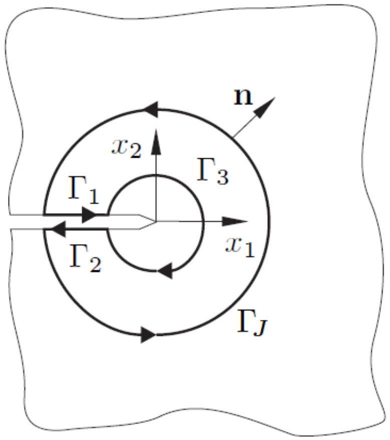
$$\frac{\partial}{\partial \mathbf{n}}\left(\mathbf{e}_p^{\omega_j}\right) = 0 \quad \text{on } \partial\omega_j \setminus (\partial\omega_j \cap \Gamma_N)$$

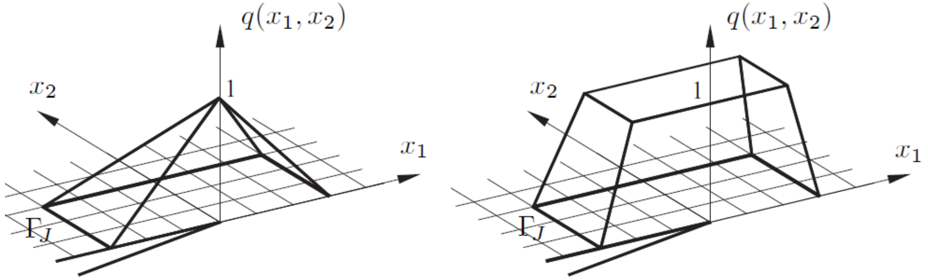
$$\frac{\partial}{\partial \mathbf{n}}\left(\mathbf{e}_p^{\omega_j}\right) = \mathcal{N}_j\left(\mathbf{t} - \boldsymbol{\sigma}\left(\mathbf{u}_p\right)\mathbf{n}\right) \quad \text{on } \partial\omega_j \cap \Gamma_N$$

$$\left[\!\left[\frac{\partial}{\partial \mathbf{n}}\left(\mathbf{e}_p^{\omega_j}\right)\right]\!\right]_\gamma = \mathcal{N}_j\left[\!\left[\frac{\partial}{\partial \mathbf{n}}\left(\mathbf{u}_p\right)\right]\!\right]_\gamma$$

Strouboulis, Zhang, Wang and Babuska, A posteriori error estimation for generalized finite element method. Computer Methods in Applied Mechanics and Engineering, 195 (2006)

Goal-oriented error estimation by configurational mechanics





$$J = \int_{\Gamma_J} \left(W dx_2 - \sigma_{ij} \mathbf{n}_j \frac{\partial u_i}{\partial x_1} \mathrm{d}\Gamma_J \right) = \int_{\Gamma_J} \left(W \delta_{1j} - \sigma_{ij} \frac{\partial u_i}{\partial x_1} \right) \mathbf{n}_j \mathrm{d}\Gamma_J$$

$$J = \int_{\Omega} \left(\sigma_{ij} \frac{\partial u_i}{\partial x_1} - W \delta_{1j} \right) \frac{\partial q}{\partial x_j} \mathrm{d}\Omega$$

$$W(x_1, x_2) = \int_0^{\varepsilon} \sigma_{ij} \mathrm{d}\varepsilon_{ij}$$

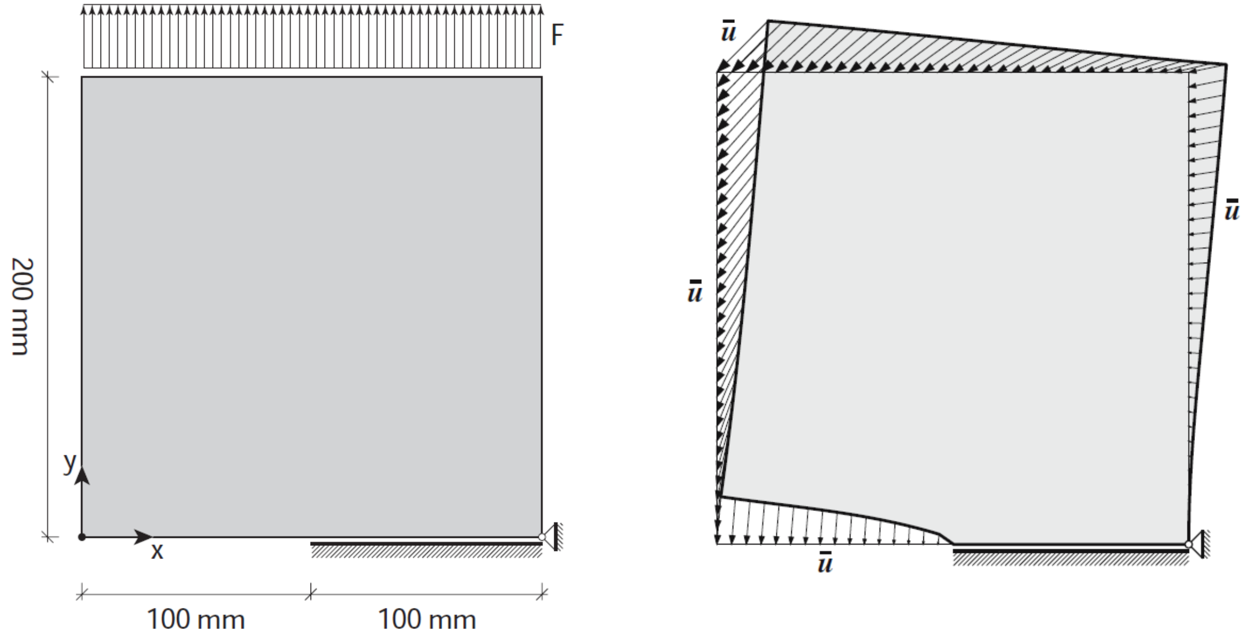
$$G = J = \int_{\Gamma_J} \left(W \delta_{1j} - \sigma_{ij} \frac{\partial u_i}{\partial x_1} \right) \mathbf{n}_j \mathrm{d}\Gamma_J = \frac{1}{E'} (K_{\mathrm{I}}^2 + K_{\mathrm{II}}^2) \qquad E' = \begin{cases} E \\ E/\left(1 - \upsilon^2\right) \end{cases}$$

Determinação do Strain Energy Release Rates via análise de sensibilidade

Giner, Fuenmayor, Besa and Tur (2002) An implementation of the stiffness derivative method as a discrete analytical sensitivity analysis and its application to mixed mode in LEFM. Engineering Fracture Mechanics

The first sensitivity of the energy functional with respect to changes in the design leads to the well-known weak form of the material or configurational force equilibrium (Materna and Barthold, 2008).

Fonte: Materna and Barthold (2008) On variational sensitivity analysis and configurational mechanics. Computational Mechanics

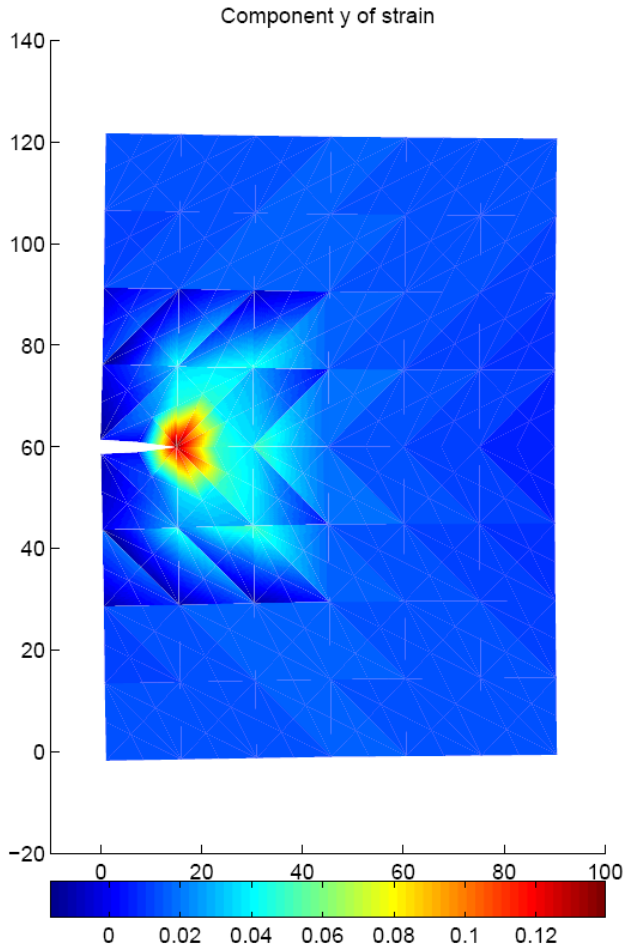


$$\Sigma_{ij} = W\delta_{ij} - \sigma_{ik}u_{k,j}$$

$$\mathscr{B}_{Eshelby}(\mathbf{u},\mathbf{v}) = \int_\Omega \Sigma(\mathbf{u}) : \nabla\, \mathbf{v}\, dV \qquad \mathscr{L}_{Eshelby}(\mathbf{v}) = \int_{\Gamma_N} \mathbf{t}_{Eshelby} \cdot \mathbf{v}\, dS$$

$$\mathscr{B}_{Eshelby}(\mathbf{u}_h,\mathbf{v}_h) \neq \mathscr{L}_{Eshelby}(\mathbf{v}_h) \quad \forall\, \mathbf{v}_h \in \mathscr{V}_h$$

Fonte: Ruter and Stein (2007) On the duality of finite element discretization error control in computational Newtonian and Eshelbian mechanics.


Component y of strain
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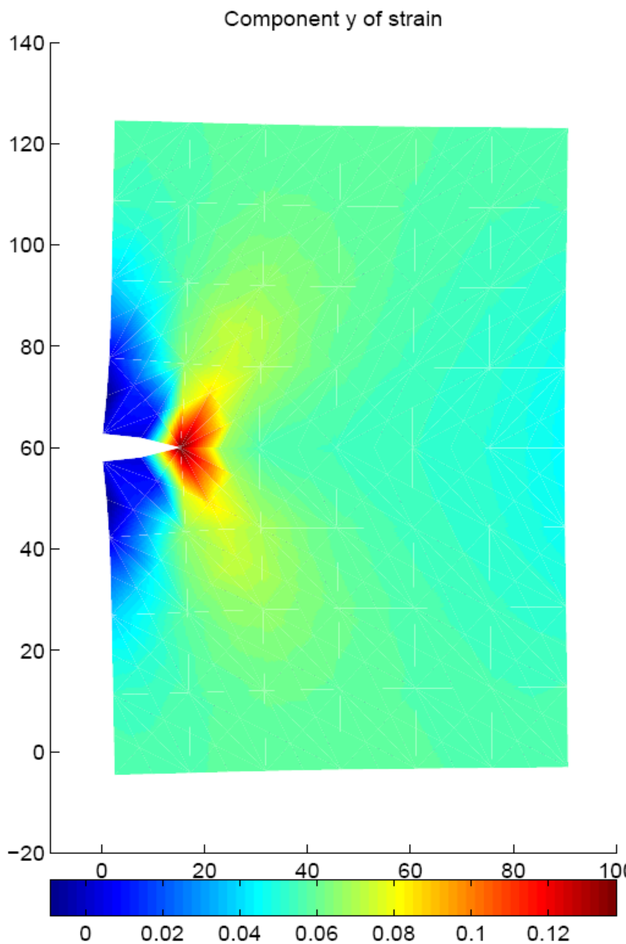
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Concluding remarks

Acknowledgements

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Thank you!