Topological optimization with local stress constraints: A review and an *Augmented Lagrangian - Level Set* approach

ACE-X 2012 - Istambul

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GRANTE
Grupo de Análise e Projeto Mecânico

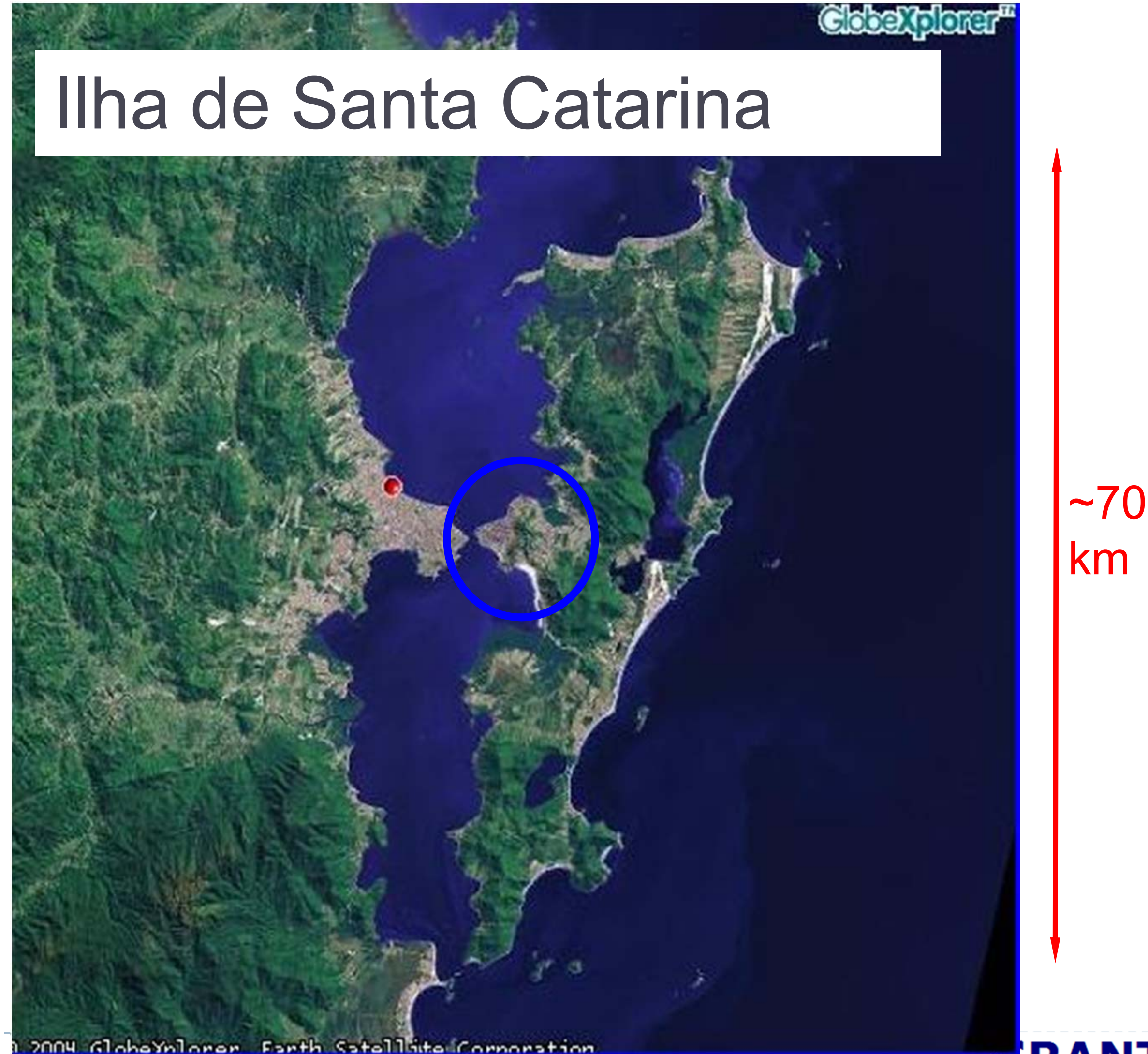
Geographical remarks

Geographical remarks

Geographical remarks

560 km

Geographical remarks



Geographical remarks

- Florianópolis

Geographical remarks

Universidade Federal de Santa Catarina

Geographical remarks

Universidade Federal de Santa Catarina UFSC

- Students: 30.000 (20.000 / 10.000)
- Teachers: 1.700
- Support: 2.500
- Courses:
  - 40 Undergraduate
  - 100 Master degree
  - 25 Doctoral degree

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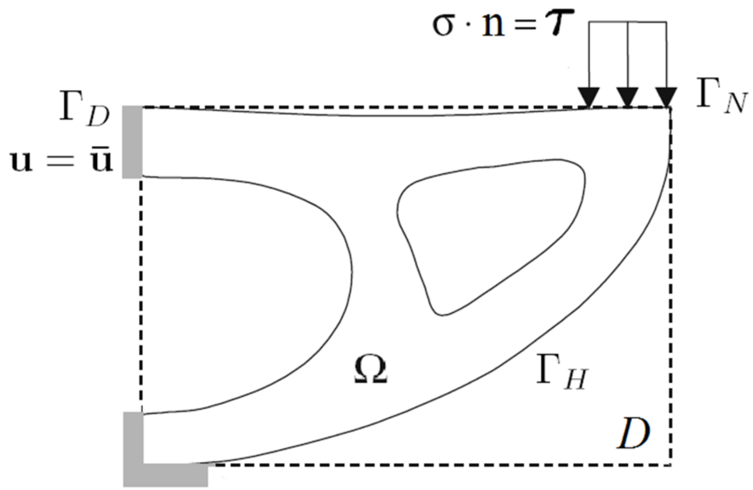
Problem context

Compliance Problem: > 80% of literature on topology optimization

- nice mathematical properties;
- single constrained problem (besides lateral constraints);
- stiffness is not necessarily a design requirement!

$$\min_{\Omega}(\text{External Work})$$

$$\int_{\Omega} d\Omega \leq V_{\max}$$



$$\int_{\Omega} \mathbf{C}\varepsilon(\mathbf{u}) \cdot \varepsilon(\mathbf{v}) \, d\Omega = \int_{\Omega} \mathbf{b} \cdot \mathbf{v} \, d\Omega + \int_{\Gamma_N} \tau \cdot \mathbf{v} \, d\Gamma$$

(linear elasticity)

Problem context

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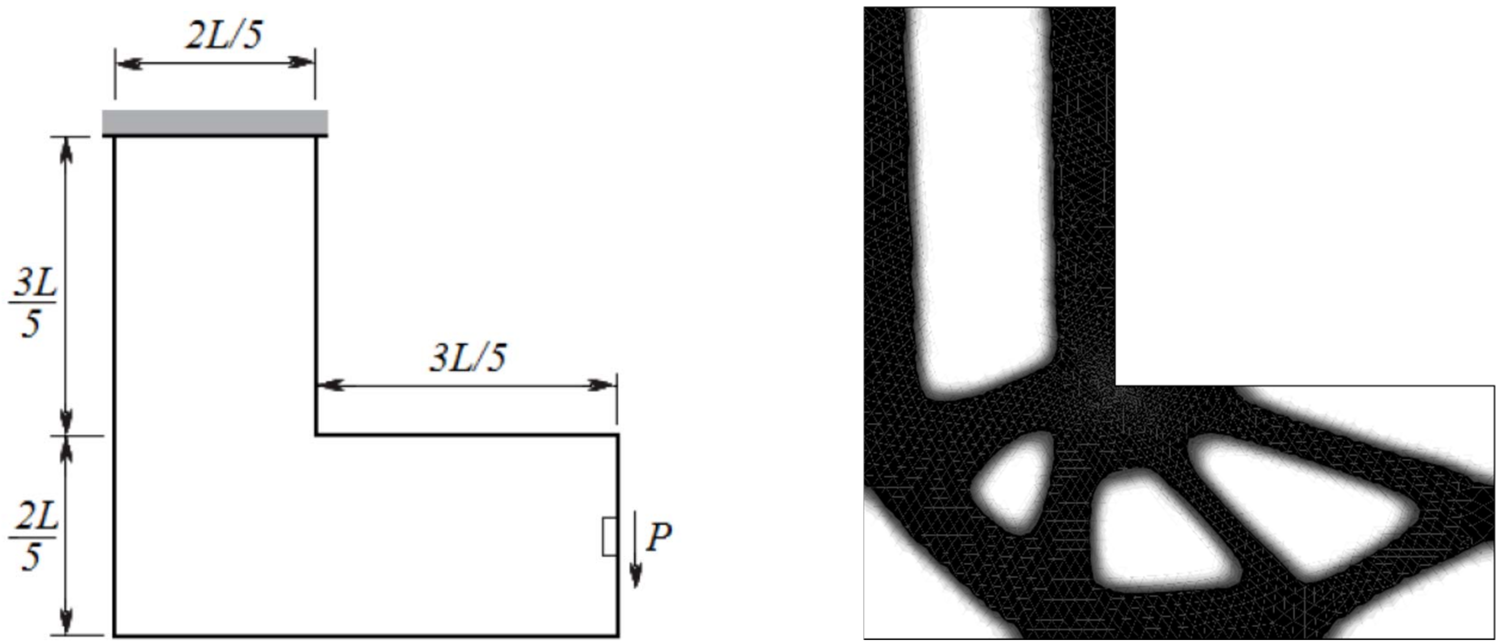

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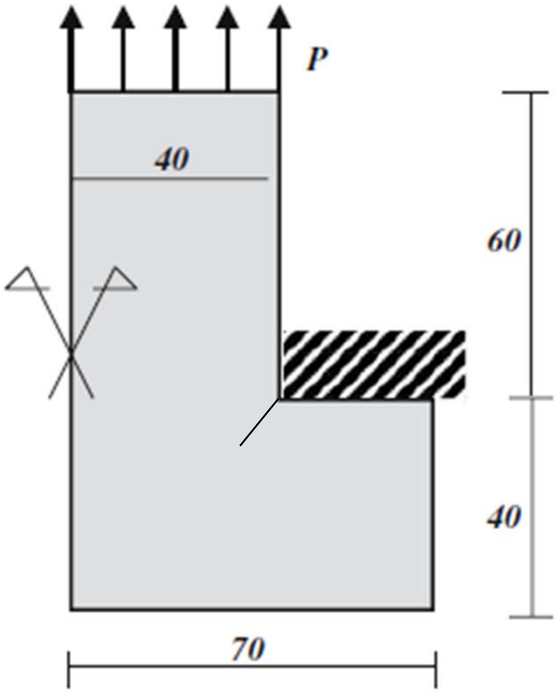
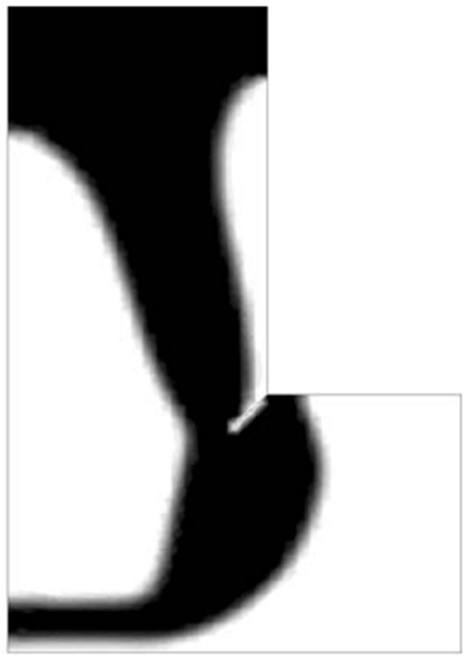




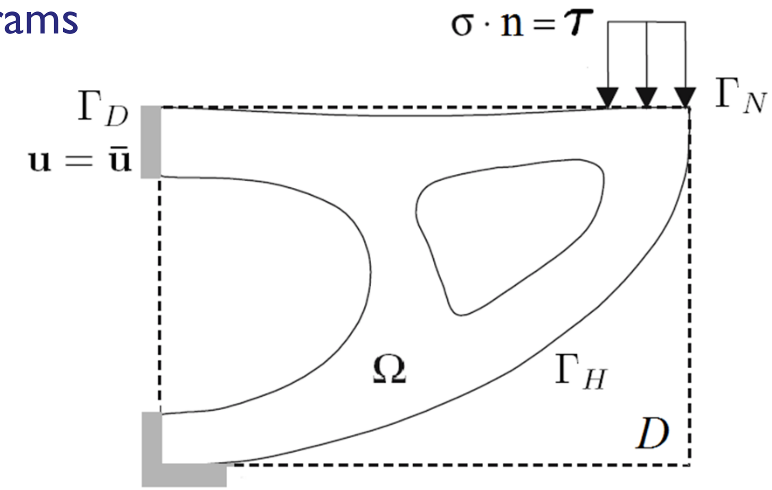
Problem context

Minimization of mass subject to material (local) failure constraints:

- common engineering requirement: lightest design that supports loads without "failure".
- much less frequent in topology optimization literature
- not consolidated in commercial programs

$$\min_{\Omega} \int_{\Omega} d\Omega$$

$$g(\sigma(x)) \leq 0 \quad \forall x \in \Omega$$



$$\int_{\Omega} \mathbf{C}\varepsilon(\mathbf{u}) \cdot \varepsilon(\mathbf{v}) \, d\Omega = \int_{\Omega} \mathbf{b} \cdot \mathbf{v} \, d\Omega + \int_{\Gamma_N} \tau \cdot \mathbf{v} \, d\Gamma$$

Problem context

Most relevant difficulties:

1. Local nature of material failure (stress) constraints

$$\hat{\sigma}(x) \leq \bar{\sigma} \quad \forall x \in \Omega$$

2. **Singularity Stress** (mathematical) phenomenon *(SIMP – Solid Isotropic Material with Penalization of Intermediate densities - approaches)* Definition of appropriate failure criterion:

3. High sensitivity of stresses to design changes

Background

Discrete (frames) structures:

Sved and Ginos (1968), Kirsch (1990), Cheng and Z. Jiang (1992), Rozvany (1991)

Continuum structures:

SIMP

1. Duysinx & Bendsoe (1998)
2. Duysinx & Sigmund(1998)

3. Fancello & Pereira (2003)
4. Pereira, Fancello & Barcellos (2004)
5. Allaire,Jouve & Maillot (2004)
6. Fancello (2006)
7. Guilherme & Fonseca (2007)
8. Bruggi (2008)
9. Bruggi & Venini (2008)
10. Paris, Navarrina, Calominas, Casteleiro (2009)

11. Le, Norato, Bruns, Ha, Tortorelli (2010)
12. Lee, James & Martins (2012)
13. Kockvara & Stingl (2012)
14. Luo & Kang (2012)

Integer

1. Svanberg & Werme (2007)

Top. Derivative

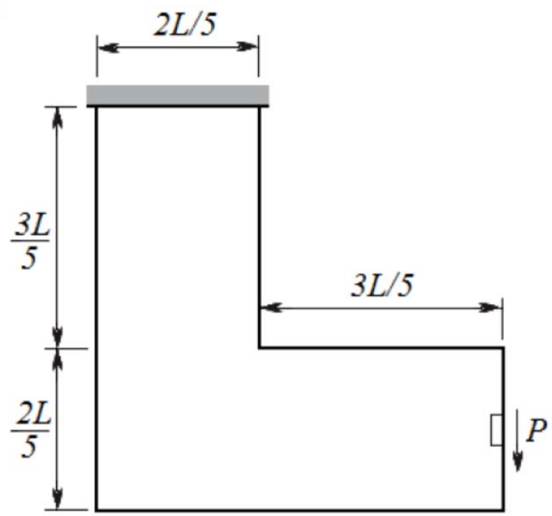
1. Amstutz, Novotny, (2010)
2. Amstutz, Novotny, de Souza Neto (2012)

Level Set

1. Allaire, Jouve, (2008)
2. Guo, Zhang, Wang, Wei (2011)
3. Xia, Shi, Liu, Wang (2012)

Background

- **Stress: objective or constraint?**
  - Min stress subject to volume constraint.
  - **Min volume subject to stress constraint**



- **Stress: local or global?**

$$\hat{\sigma}(x) \leq \bar{\sigma} \quad \forall x \in \Omega$$

$$\hat{\sigma}_e \leq \bar{\sigma} \quad e = 1, 2 \ldots n$$

$$\max_{x \in \Omega}\{\hat{\sigma}(x)\} \leq \bar{\sigma}$$

$$\max_{e}\{\hat{\sigma}_e\} \leq \bar{\sigma} \quad e = 1, 2 \ldots n$$

$$\left(\int_\Omega \hat{\sigma}(x)^P \, d\Omega\right)^{1/P} \leq \bar{\sigma}_\Omega$$

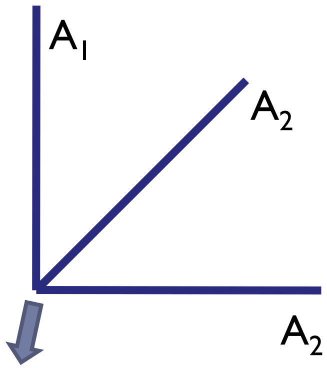
$$\left(\sum_{e=1}^{n} \Omega_e \hat{\sigma}_e^P\right)^{1/P} \leq \bar{\sigma}_\Omega$$
$$e = 1, 2 \ldots n$$

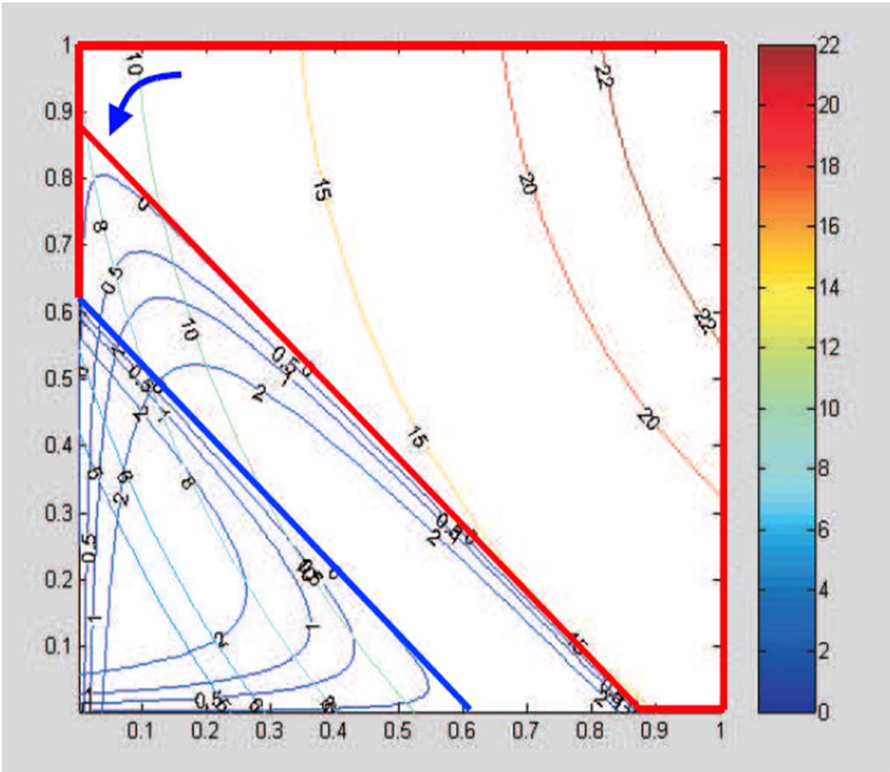
Background

Singularity Stress

- Firstly seen in discrete (frame) structures:

  Sved and Ginos (1968), Kirsch (1990), Cheng and Z. Jiang (1992),





- *Cheng & Guo (1997),*

  Epsilon-regularization

Background

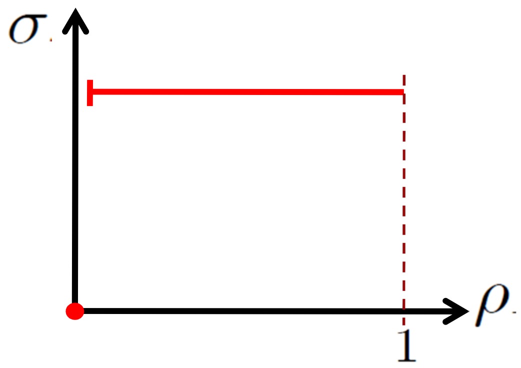
Singularity Stress

- Continuum structures: *Duysinx and Bensoe (1998)*
  - Local stress criterion based on failures in microstructures
  - Extension to SIMP approach:

$$\langle \sigma \rangle = \rho E \langle \varepsilon \rangle \quad \rho = A/2$$

$$\sigma = \frac{\langle \sigma \rangle}{\rho}$$

$$\boxed{\sigma = E \langle \varepsilon \rangle \leq \bar{\sigma}}$$



$$\langle \sigma_{ij} \rangle = \langle E_{ijkl}(\rho) \rangle \langle \varepsilon_{ij} \rangle \qquad \langle E(\rho) \rangle = \rho^p E$$

$$\sigma_{ij} = \frac{\langle \sigma_{ij} \rangle}{\rho^q} \qquad \sigma_{ij} = \frac{\rho^p}{\rho^q} E_{ijkl} \langle \varepsilon_{ij} \rangle \qquad \Rightarrow p = q$$

Background

Singularity Stress

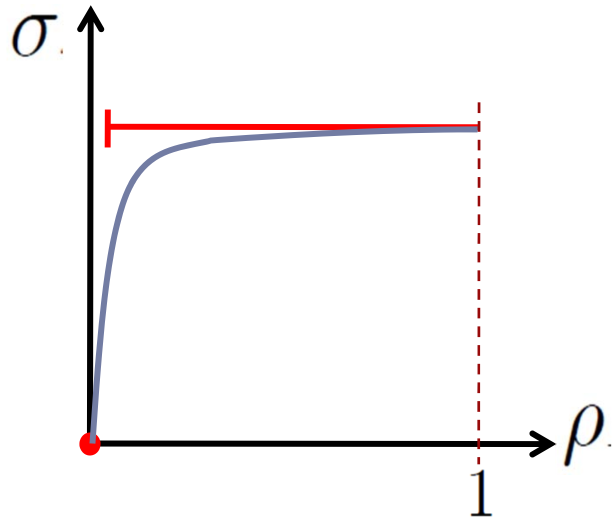
- Epsilon-Regularization *Cheng & Guo (1997)*

$$\rho\left(\langle\|\sigma\|\rangle/\sigma_y - 1\right) \leq \varepsilon \qquad \varepsilon^2 = \rho_{\min} \leq \rho$$

- *q-p* regularization *(Bruggi 2008)*

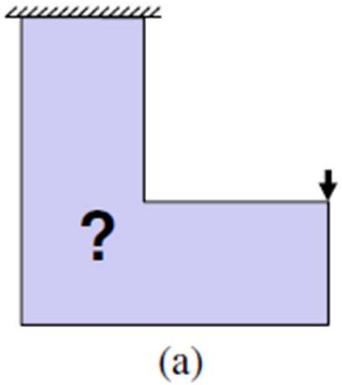
$$\sigma_{ij} = \frac{\rho^p}{\rho^q} E_{ijkl}\left\langle \varepsilon_{ij} \right\rangle \qquad q < p$$

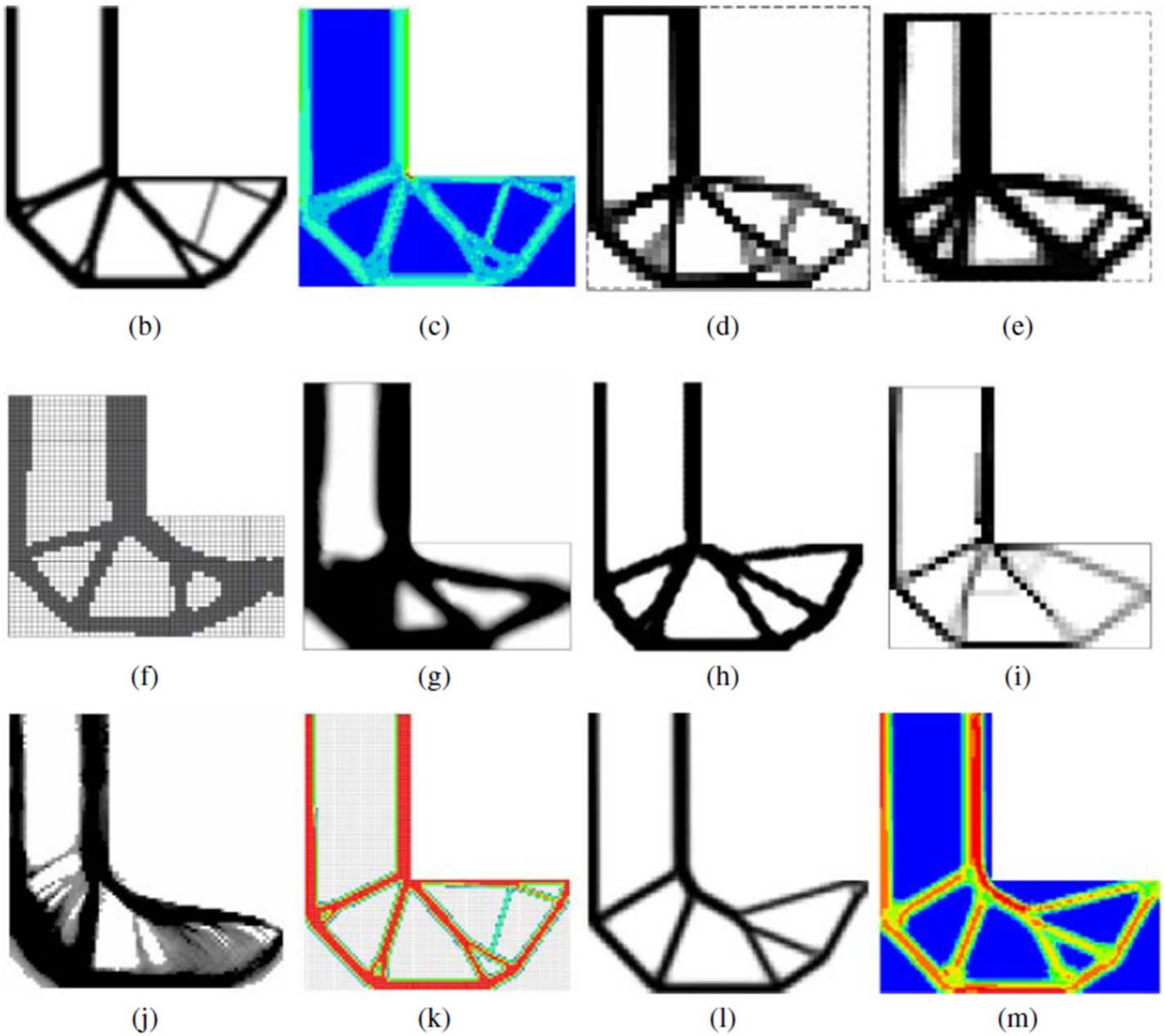
- … *other alternatives*



Background

Le, Norato, Bruns, Ha, Tortorelli (2010)

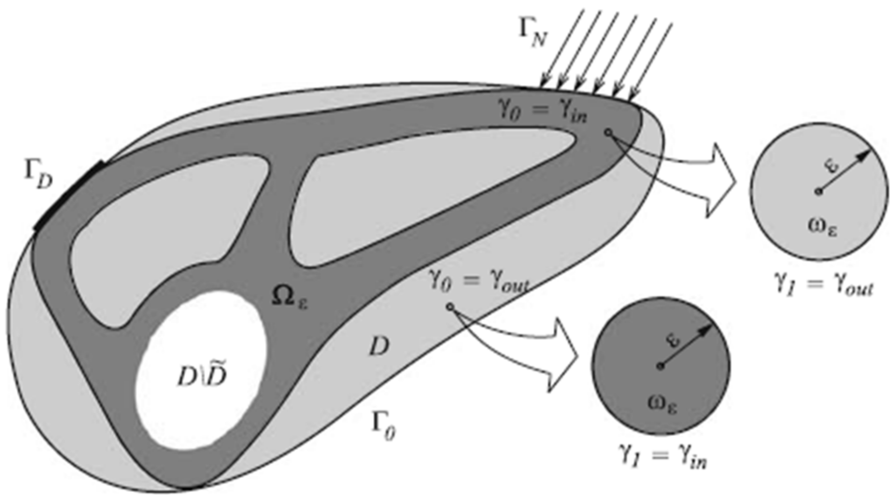


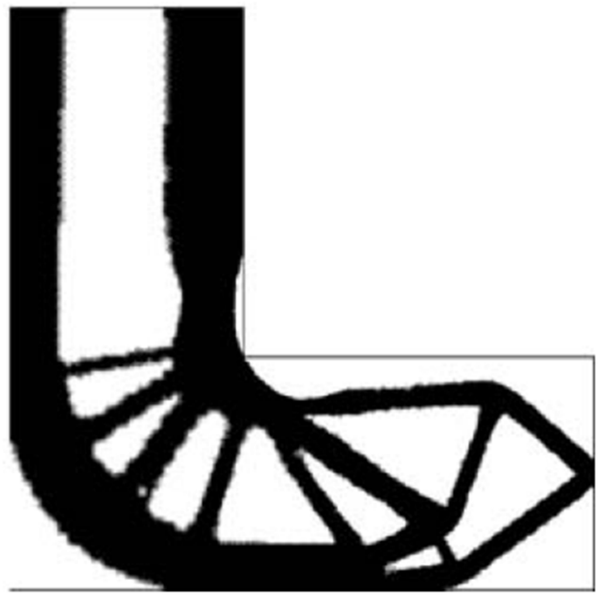


color scale ranges). **a** Problem definition, **b** Min. compliance design, **c** Stress distribution in **b**, **d** Duysinx and Bendsøe (1998), **e** Duysinx and Sigmund (1998), **f** Svanberg and Werme (2007), **g** Pereira et al. (2004), **h** Bruggi and Venini (2008), **i** París et al. (2009), **j** Guilherme and Fonseca (2007), **k** Altair Optistruct (2007) (density distribution), **l** Present work, **m** Stress distribution in **l**

Background

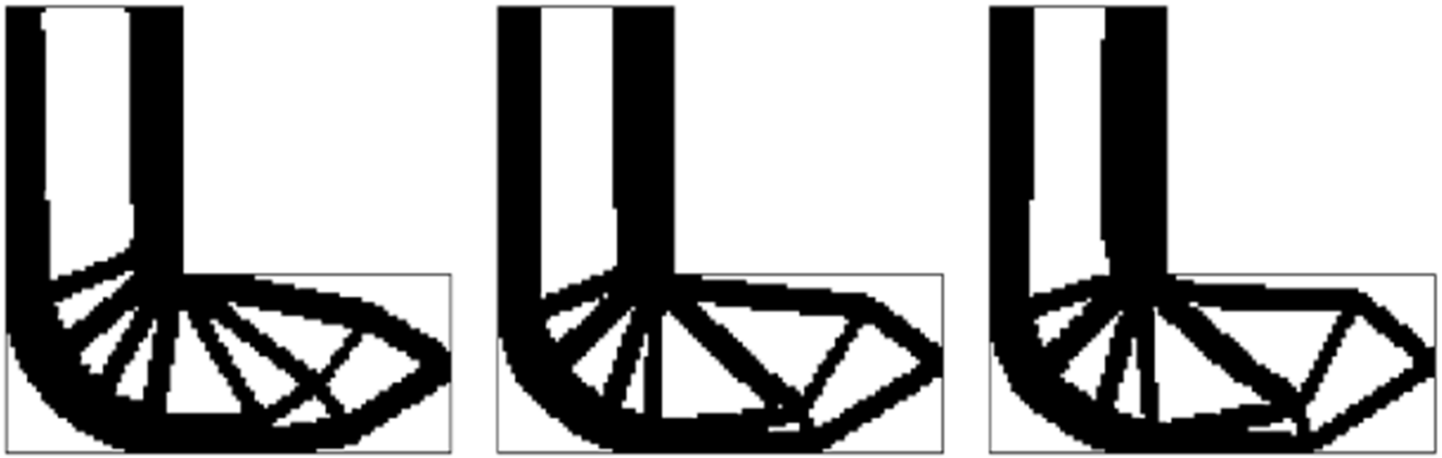
- Amstutz & Novotny, 2010, 2012 → **Topological derivative**





Level set method

- *Sethian & Wiegmann, 2000* (*"Structural boundary design via level set and immersed interface methods"*)
- *Wang, Wang & Guo (2003) and Allaire, Jouve & Toader (2004)*: Sensitivity analysis and velocities at the boundary
- *Allaire, Jouve & Toader (2008)* Minimum stress optimal design with the level set method



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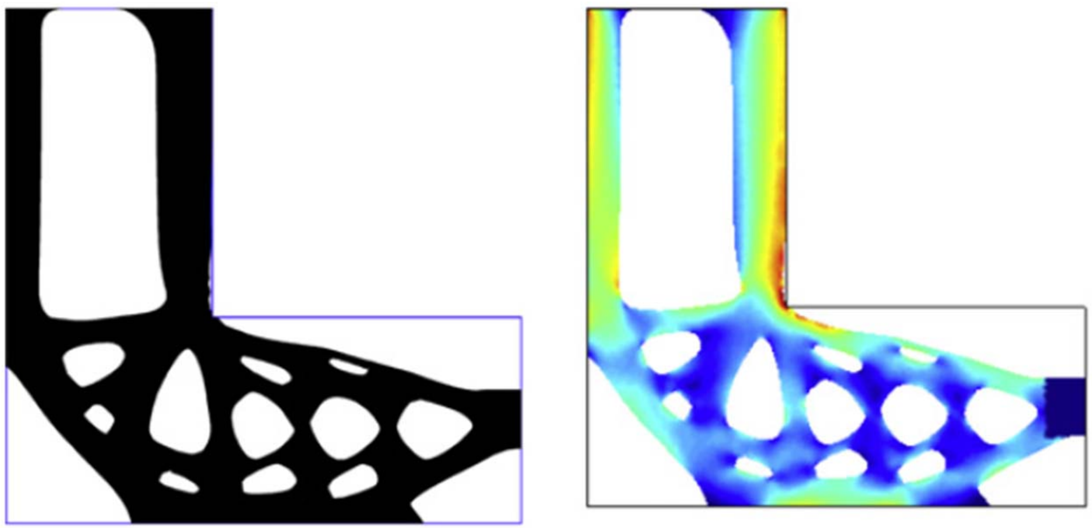
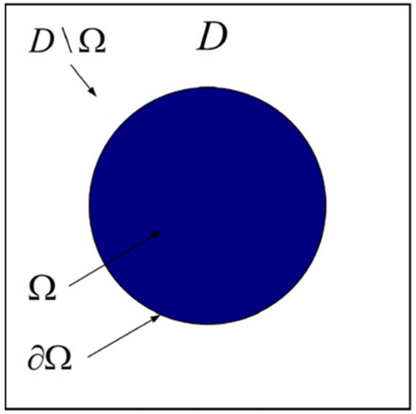
- *Xia, Shi, Liu, Wang (2012)* A level set solution to the stress-based structural shape and topology optimization



Level set method

- Variation of the boundary by a function Ø : boundary = zero-level set of function Ø.

$$\begin{cases} \phi(\mathbf{x}) > 0 & \forall \mathbf{x} \in \Omega \setminus \partial\Omega \quad \text{(região com material)}, \\ \phi(\mathbf{x}) = 0 & \forall \mathbf{x} \in \partial\Omega \quad \text{(na fronteira)}, \\ \phi(\mathbf{x}) < 0 & \forall \mathbf{x} \in D \setminus \Omega \quad \text{(região sem material)}. \end{cases}$$

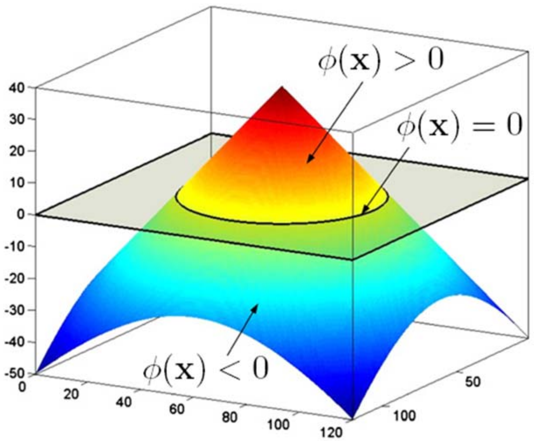


- Spatial description of zero-level set

$$\phi(\mathbf{x}(t), t) = 0 \quad \forall t,\ \mathbf{x} \in \partial\Omega(t)$$

- Material derivative

$$\frac{d\phi(\mathbf{x}(t), t)}{dt} = \frac{\partial\phi(\mathbf{x}(t), t)}{\partial t} + \nabla\phi(\mathbf{x}(t), t) \cdot \mathbf{V} = 0$$



- *Hamilton Jacobi Equation*

$$\frac{\partial\phi(\mathbf{x}(t), t)}{\partial t} - v_n \|\nabla\phi(\mathbf{x}(t), t)\| = 0 \qquad v_n = \mathbf{V} \cdot \mathbf{n}$$

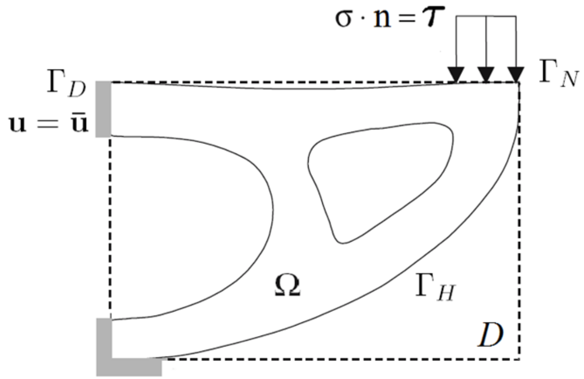
Level set method

- *Problem Statement I*

$$\min_{\Omega} \int_{\Omega} d\Omega$$

$$g(\sigma(x)) \leq 0 \quad \forall x \in \Omega$$

$$a_{\Omega}(\mathbf{u},\mathbf{v}) = l_{\Omega}(\mathbf{v}) \quad \forall \mathbf{v} \in V$$



- Integral in Ω → integral in D by Heaviside function

$$H(\phi) = \begin{cases} 0 & \text{if } \phi < 0, \\ 1 & \text{if } \phi \geq 0, \end{cases}$$

$$\rho(\phi) = \rho_1 H(\phi) + \rho_2(1 - H(\phi)) \qquad \rho_1 >> \rho_2$$

$$\mathbf{C}(\phi) = \mathbf{C}_1 H(\phi) + \mathbf{C}_2(1 - H(\phi)) \quad E_1 >> E_2$$

$$a_{\phi}(\mathbf{u},\mathbf{v}) = l_{\phi}(\mathbf{v}) \quad \forall \mathbf{v} \in V$$

$$a_{\phi}(\mathbf{u},\mathbf{v}) = \int_D \mathbf{C}(\phi)\varepsilon(\mathbf{u}) \cdot \varepsilon(\mathbf{v})\, dD$$

$$l_{\phi}(\mathbf{v}) = \int_D H(\phi)\mathbf{b} \cdot \mathbf{v}\; d\Omega + \int_{\Gamma_N} \tau \cdot \mathbf{v}\; d\Gamma$$

Level set method

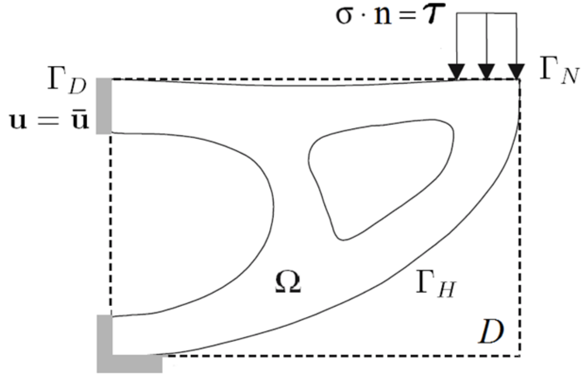
- *Problem Statement 2*

$$\min_{\phi} \int_D \rho(\phi)\, dD$$

$$a_\phi(\mathbf{u},\mathbf{v}) = l_\phi(\mathbf{v}) \quad \forall \mathbf{v} \in V$$

$$g(\mathbf{u}, H(\phi)) = \frac{\sigma_{vM}(\mathbf{u}, H(\phi))}{\sigma_{adm}} - 1 \leq 0, \quad \forall \mathbf{x}$$

$$\sigma_{vM}(\mathbf{u}, H(\phi)) = H(\phi)^q \sigma_{vM}$$



- *Problem Statement 3: **Augmented Lagrangian***

$$\min_{\phi} J(\phi) = \int_D \rho(\phi) dD + \int_D \alpha h(\mathbf{u}, H(\phi)) dD + \int_D \tfrac{c}{2} h^2(\mathbf{u}, H(\phi)) dD$$

$$a_\phi(\mathbf{u},\mathbf{v}) = l_\phi(\mathbf{v}) \quad \forall \mathbf{v} \in V$$

$$h(\mathbf{u}, H(\phi)) = \max\left\{ g(\mathbf{u}, H(\phi)); -\frac{\alpha}{c} \right\}$$

Level set method

- *Sensitivity analysis*

$$\frac{dJ}{d\phi}[\delta\phi] = \int_D G(\phi)\delta(\phi)\delta\phi dD$$

$$G(\phi) = \begin{cases} \Delta\rho + qH(\phi)^{q-1}\dfrac{\sigma_{vM}}{\sigma_{adm}}\left[\alpha + cg(\mathbf{u}, H(\phi))\right] \\ \qquad + \Delta\mathbf{C}\varepsilon(\mathbf{u})\cdot\varepsilon(\boldsymbol{\lambda}) - \mathbf{b}\cdot\boldsymbol{\lambda} & \text{if } g(\mathbf{u}, H(\phi)) \geq -\dfrac{\alpha}{c} \\ \Delta\rho + \Delta\mathbf{C}\varepsilon(\mathbf{u})\cdot\varepsilon(\boldsymbol{\lambda}) - \mathbf{b}\cdot\boldsymbol{\lambda}, & \text{if } g(\mathbf{u}, H(\phi)) < -\dfrac{\alpha}{c} \end{cases}$$

- *Adjoint equation*

$$\int_D \mathbf{C}(\phi)\varepsilon(\boldsymbol{\lambda})\cdot\varepsilon(\delta\mathbf{u})dD = -\int_D \left[\alpha + ch(\mathbf{u}, H(\phi))\right]\mathbf{C}_1\mathbf{A}(\mathbf{u}, \phi)\cdot\varepsilon(\delta\mathbf{u})dD$$

$$\mathbf{A}(\mathbf{u}, \phi) = \begin{cases} \dfrac{3}{2}\dfrac{H(\phi)^q}{\sigma_{adm}\sigma_{vM}}\mathbf{P}^T\mathbf{S}(\mathbf{u}) & \text{if } g(\mathbf{u}, H(\phi)) \geq -\dfrac{\alpha}{c} \\ \mathbf{0}, & \text{if } g(\mathbf{u}, H(\phi)) < -\dfrac{\alpha}{c} \end{cases}$$

Level set method

- *Sensitivity analysis → velocity of the boundary*

$$J(\phi + t\delta\phi) = J(\phi) + t\frac{dJ}{d\phi}[\delta\phi] + \vartheta(t^2) \qquad \delta\phi = \frac{\partial\phi}{\partial t} = v_n\|\nabla\phi\|$$

- ***The objective function decreases its value for a normal velocity of the boundary:***

$$\boxed{v_n = -G(\phi)} \qquad J(\phi + t\delta\phi) = J(\phi) - t\int_D G^2(\phi)\delta(\phi)\|\nabla\phi\|dD + \vartheta(t^2)$$

- Upwind schemes for the discrete solution of Hamilton Jacobi equation

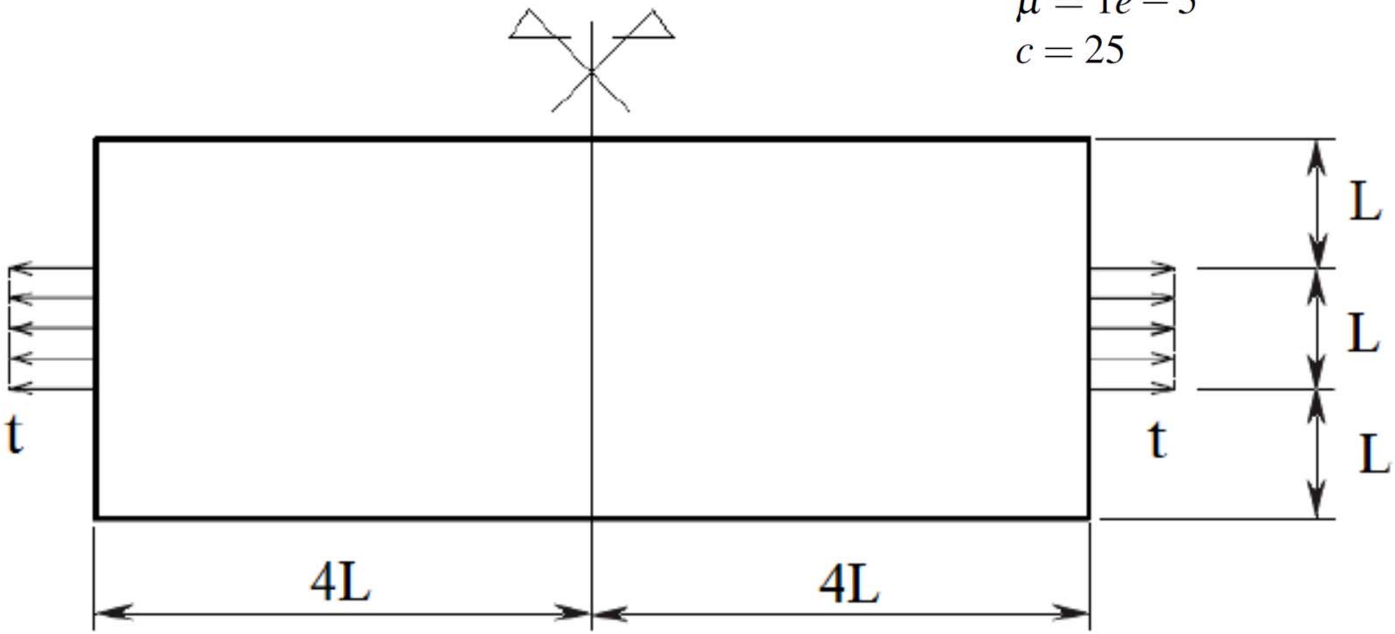
$$\phi_i^{n+1} = \phi_i^n - \Delta t\left[\max\left((v_n)_i, 0\right)\nabla_i^+ + \min\left((v_n)_i, 0\right)\nabla_i^-\right]$$

- Courant-Friedrich-Lewy (CFL) condition
- Function *Ø* → *"distance function with signal"* → **Reinicialization** (bad stuff)

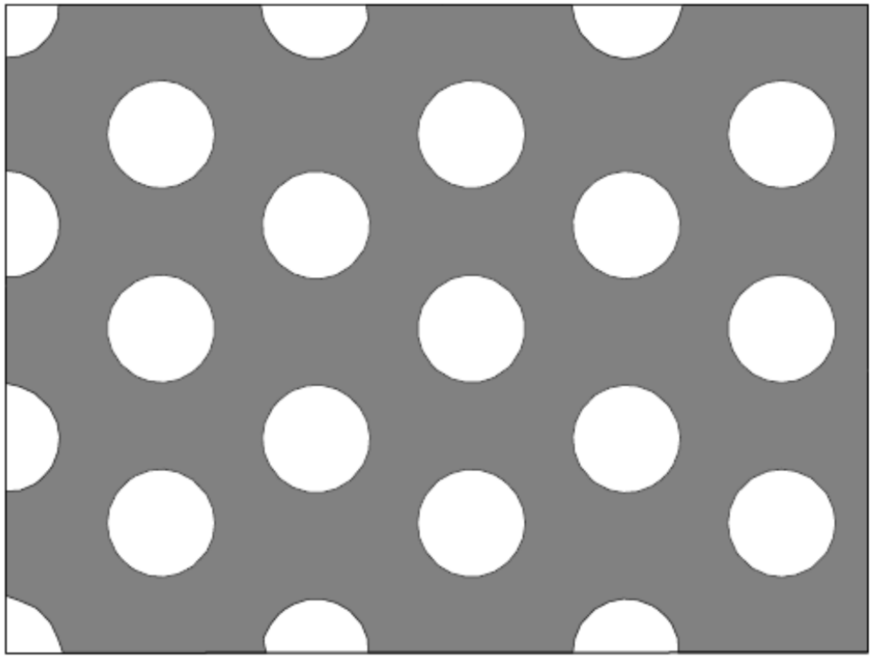
Numerical Tests

Example 1: Traction of a Bar

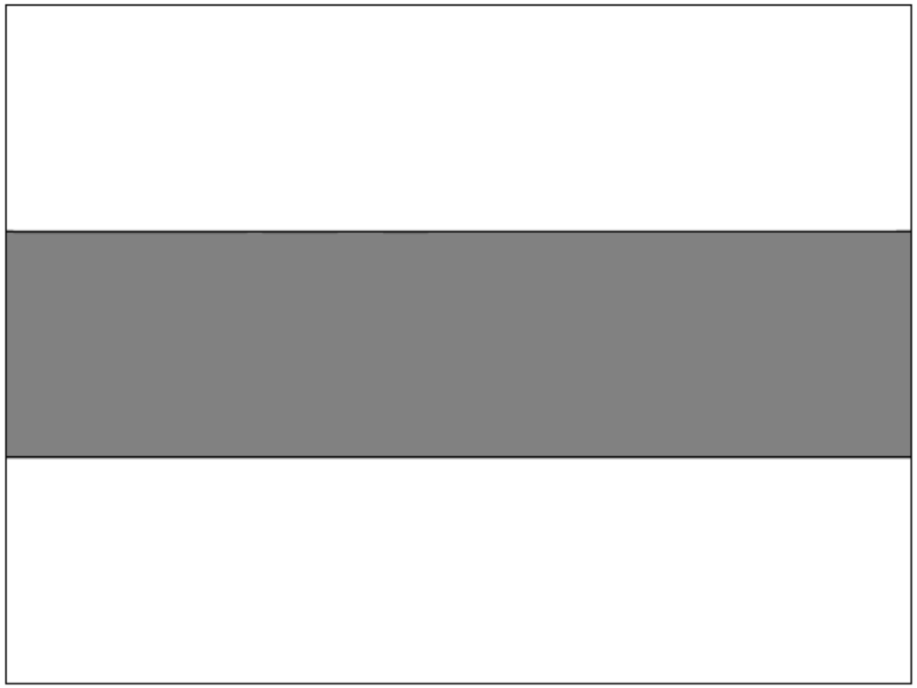
$t = 1,0\ Pa$
$\sigma_{adm} = 1,0\ Pa$
$L = 1,0\ m$
Elements mesh: 60 x 45
Grid level set: 61 x 46
$\xi = 0.7 \min(a,b)$
$\mu = 1e-5$
$c = 25$



Numerical Tests



Numerical Tests



Numerical Tests

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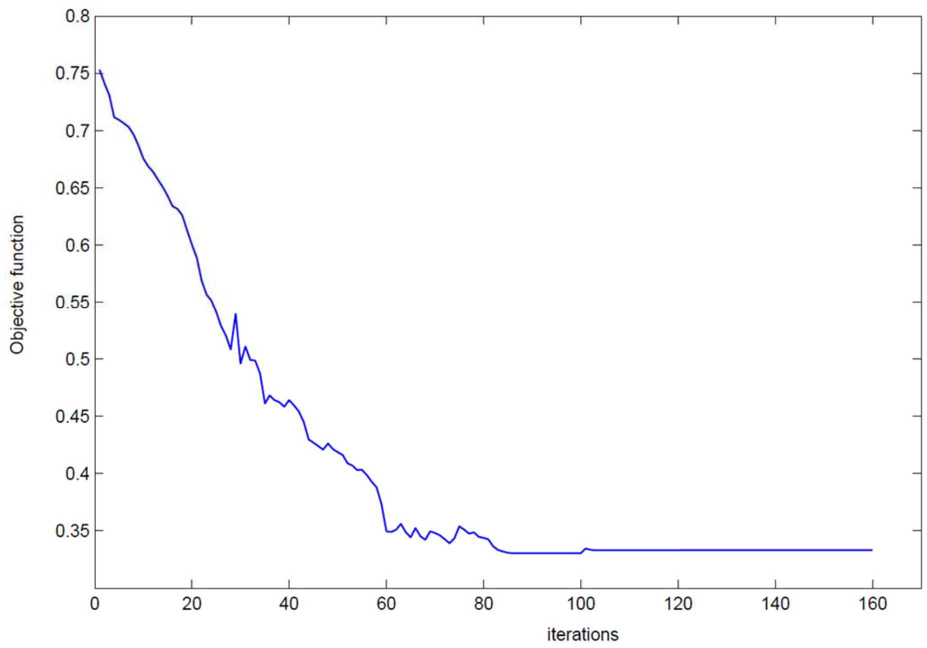
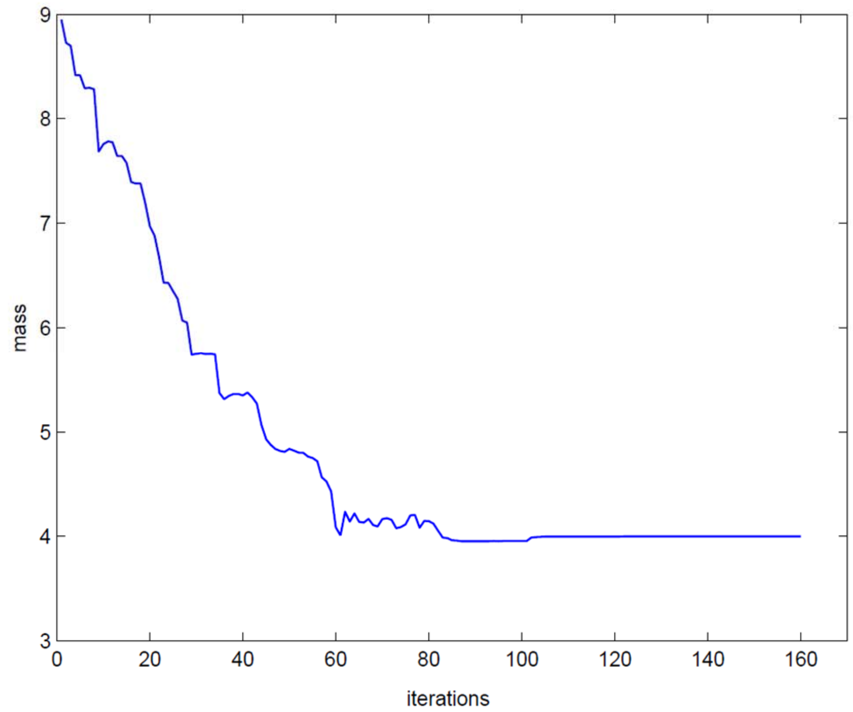
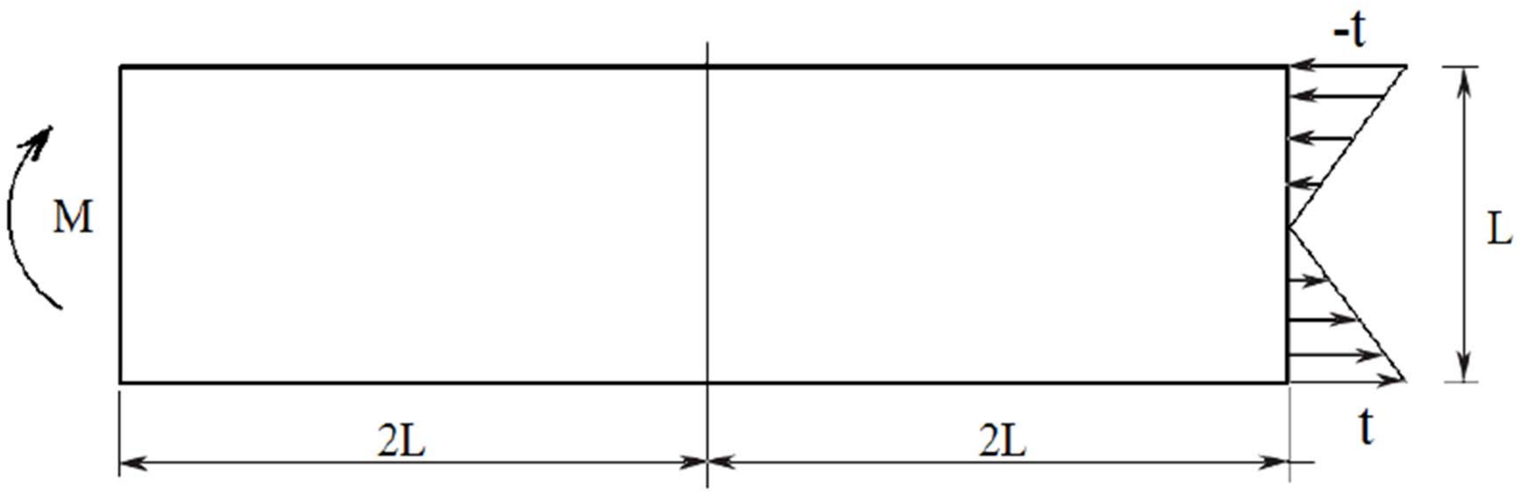




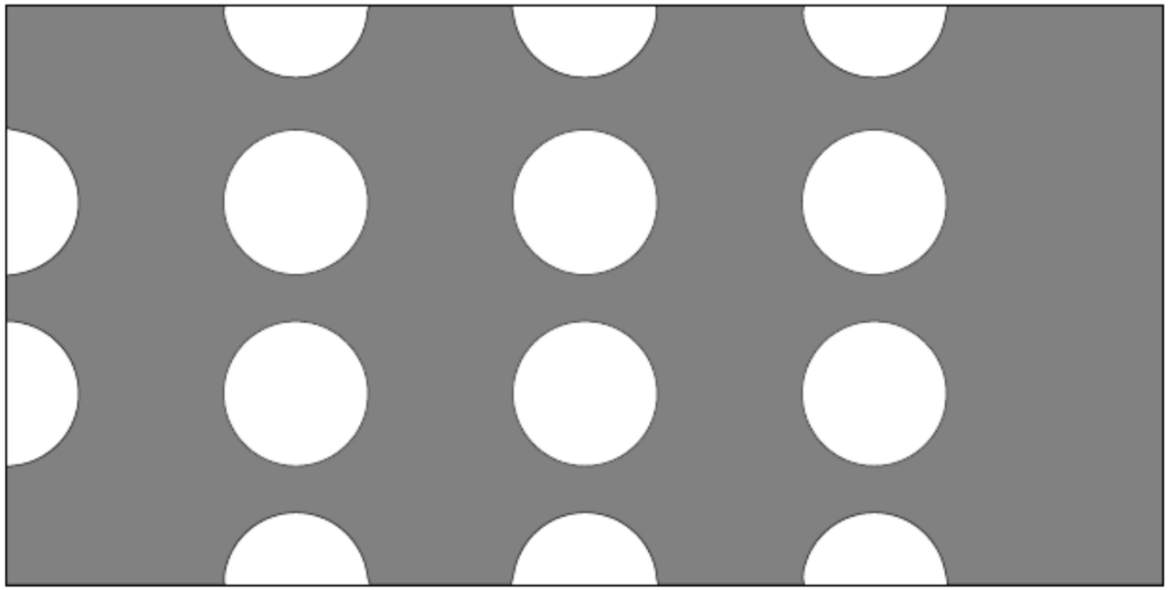
Numerical Tests

Example 2: Bending of a bar

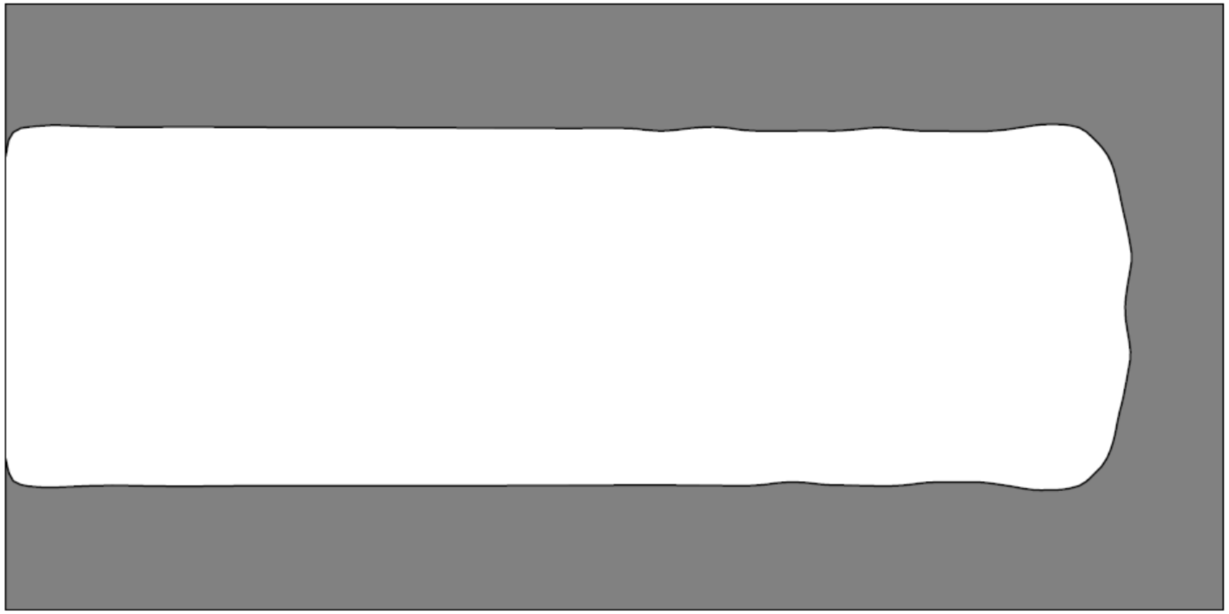
$t = 30\ Pa$
$\sigma_{adm} = 35\ Pa$
$L = 10\ m$
Elements mesh: 160 x 80
Grid level set: 161 x 81
$\xi = 0.7 \min(a, b)$
$\mu = 1e-5$
$c = 500$



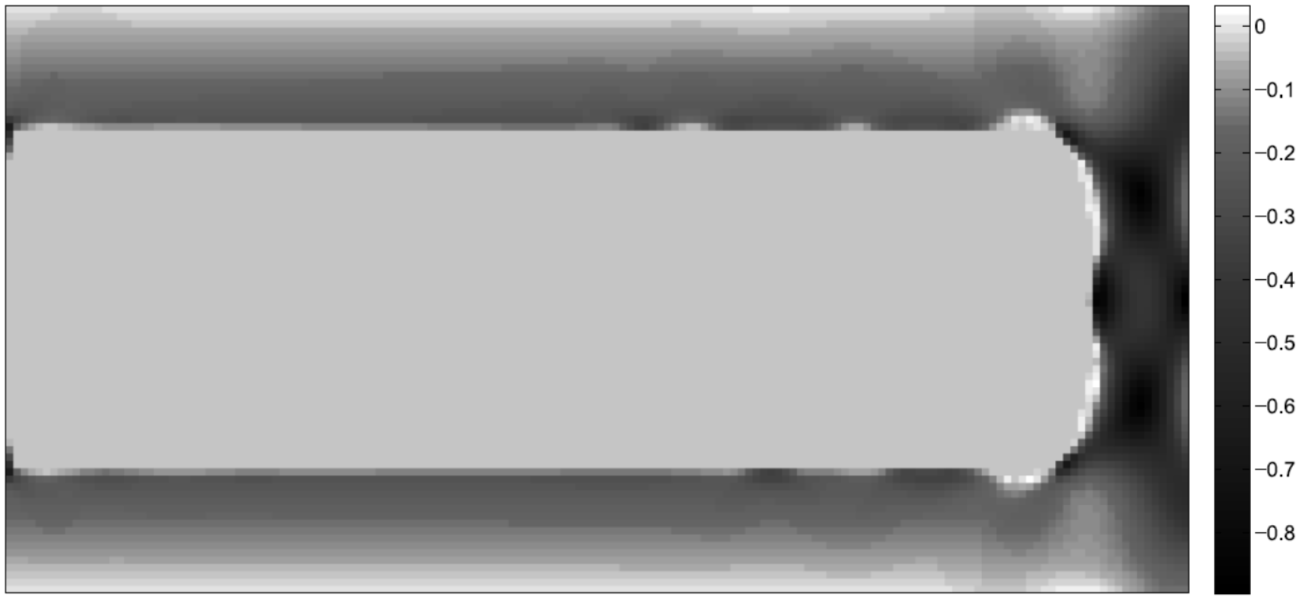
Numerical Tests



Numerical Tests



Numerical Tests



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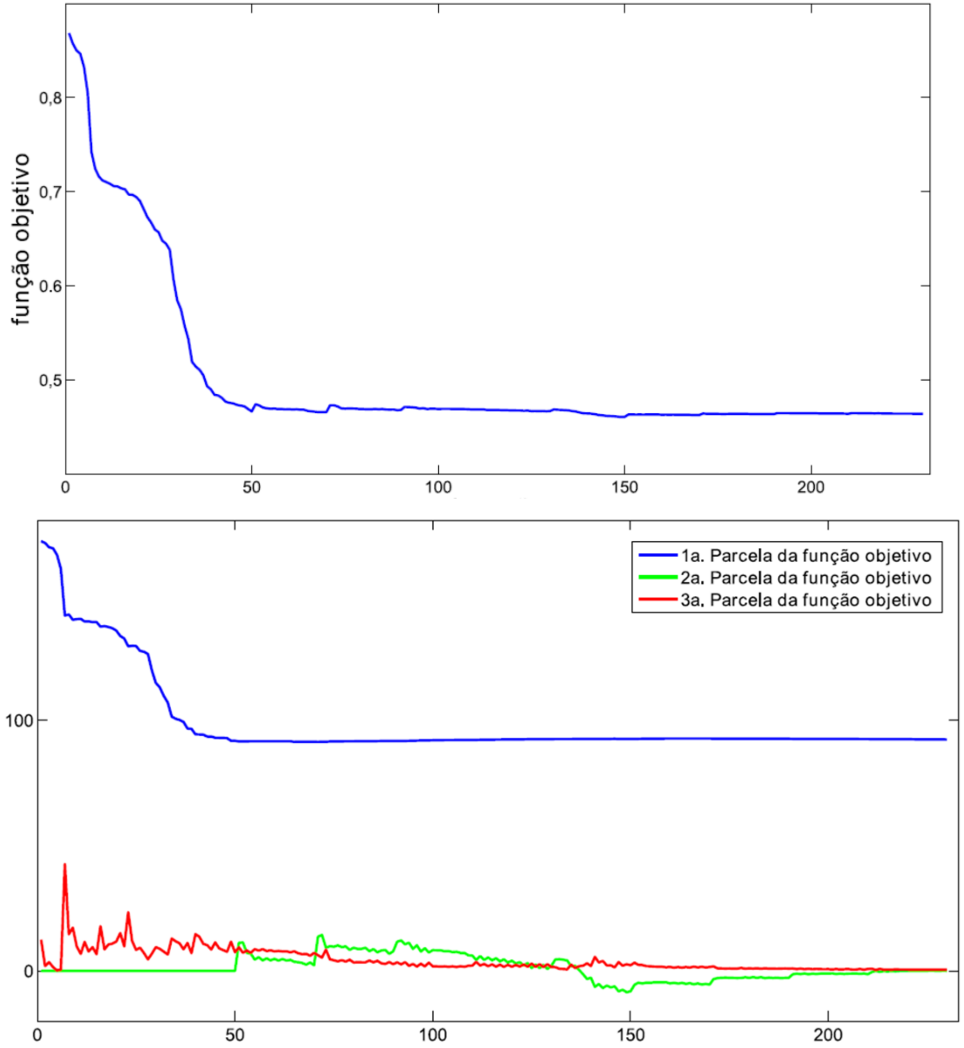
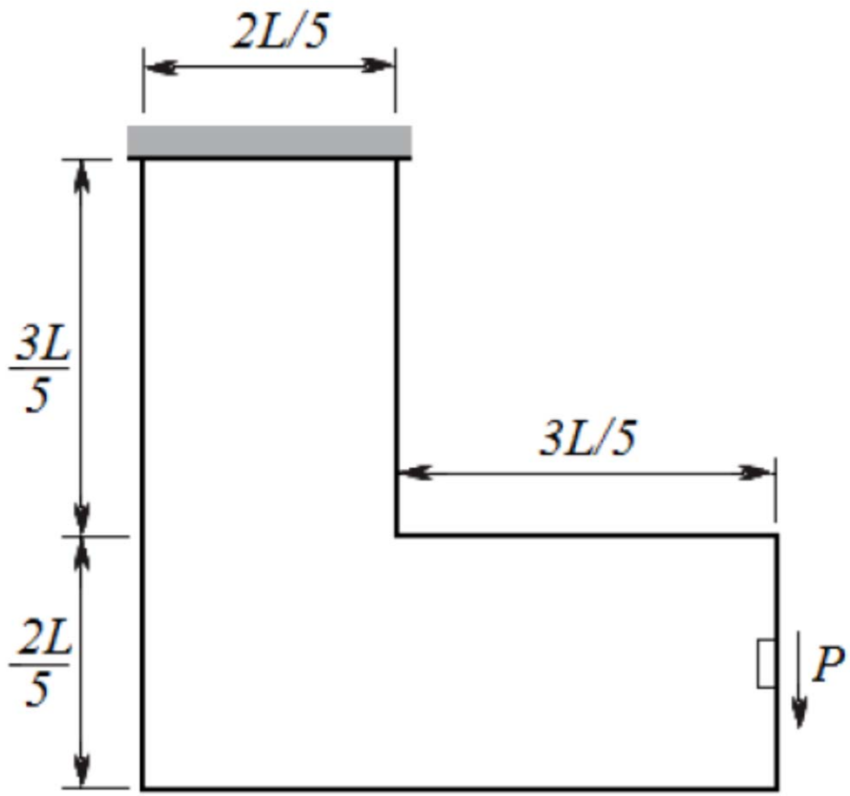


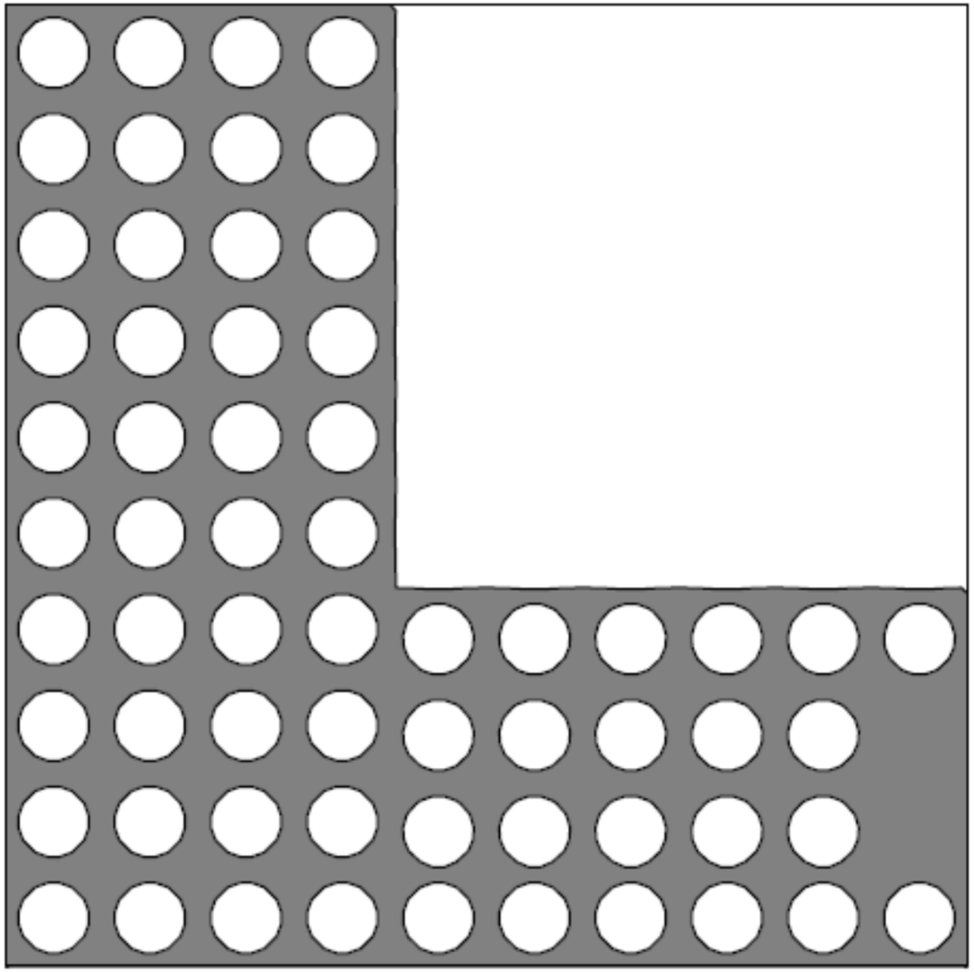
Numerical Tests

Example 3: L-shaped domain

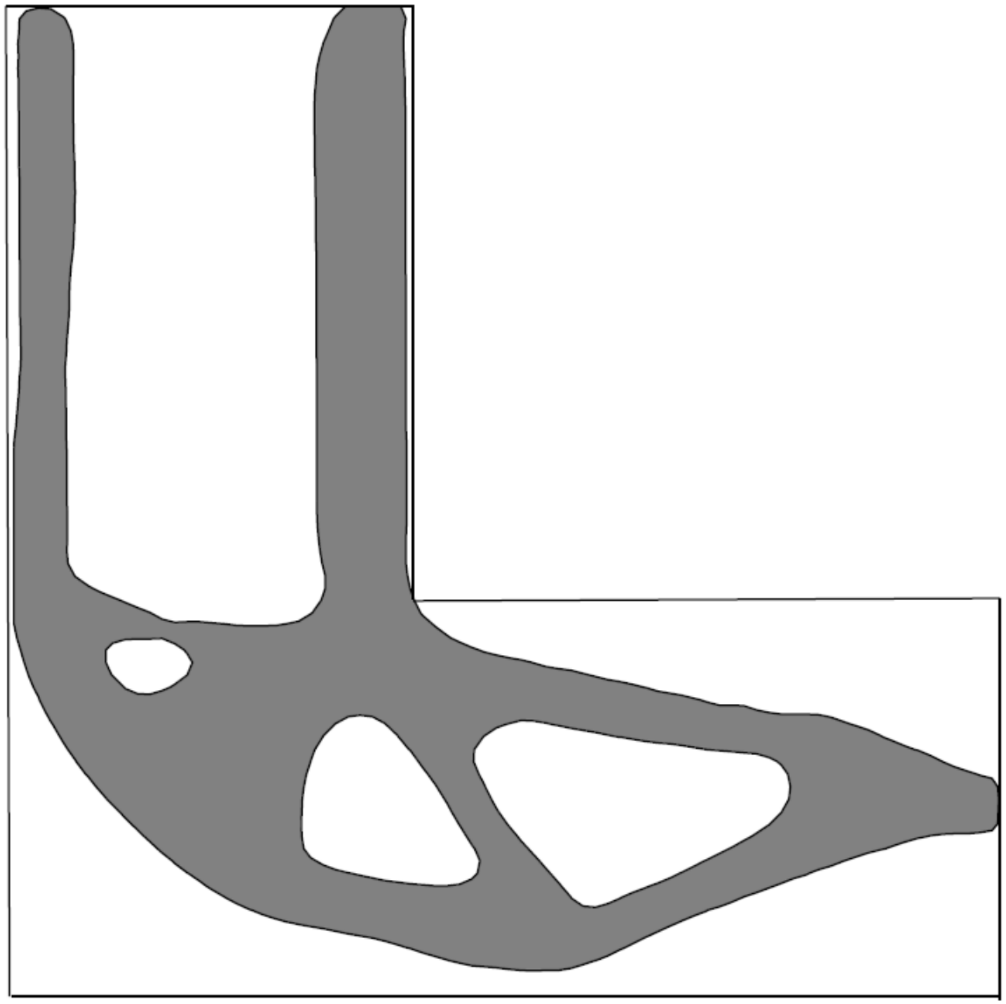
$P = 1,0\,N$
$\sigma_{adm} = 42\,Pa$
$L = 1,0\,m$
Elements mesh: 80 x 80
Grid level set: 81 x 81
$\xi = 0.7\min(a,b)$
$\mu = 1e-6$
$c = 410$



Numerical Tests



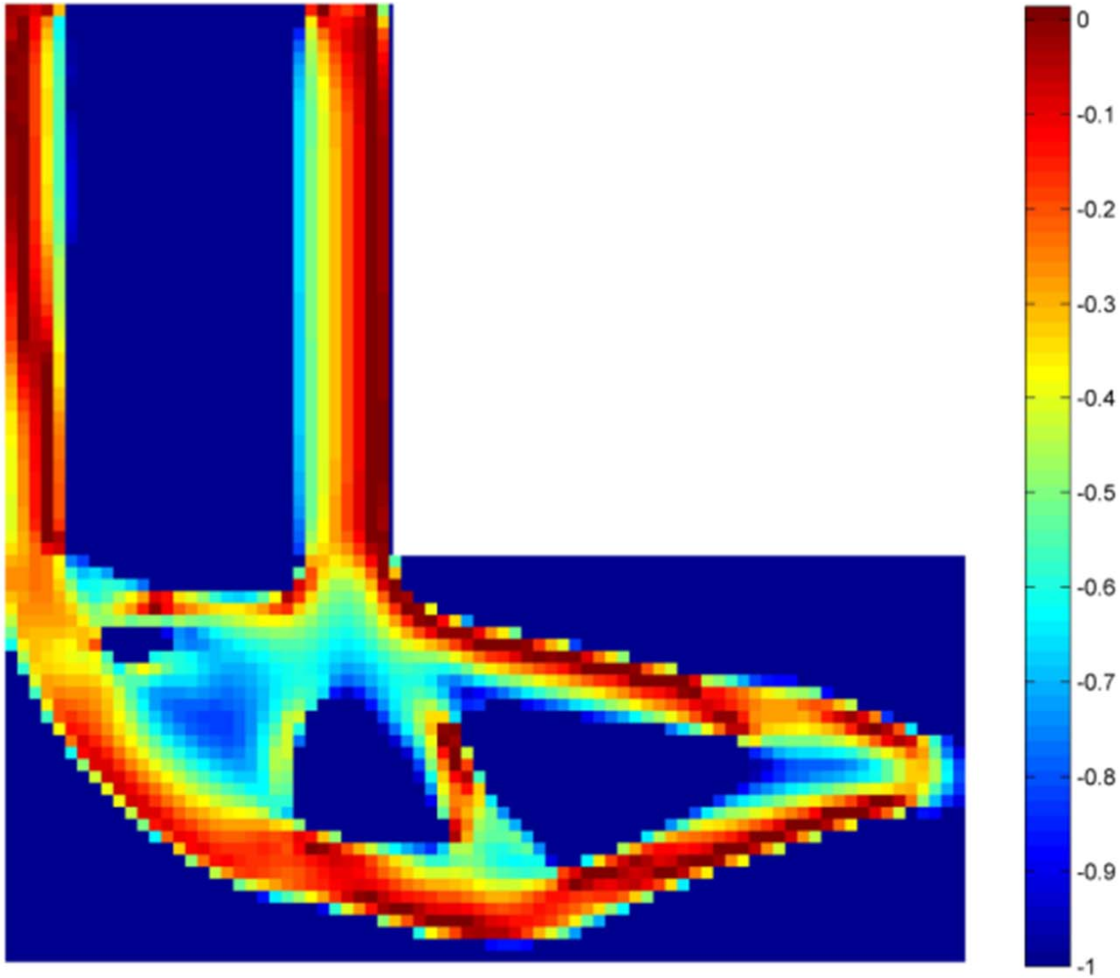
Numerical Tests



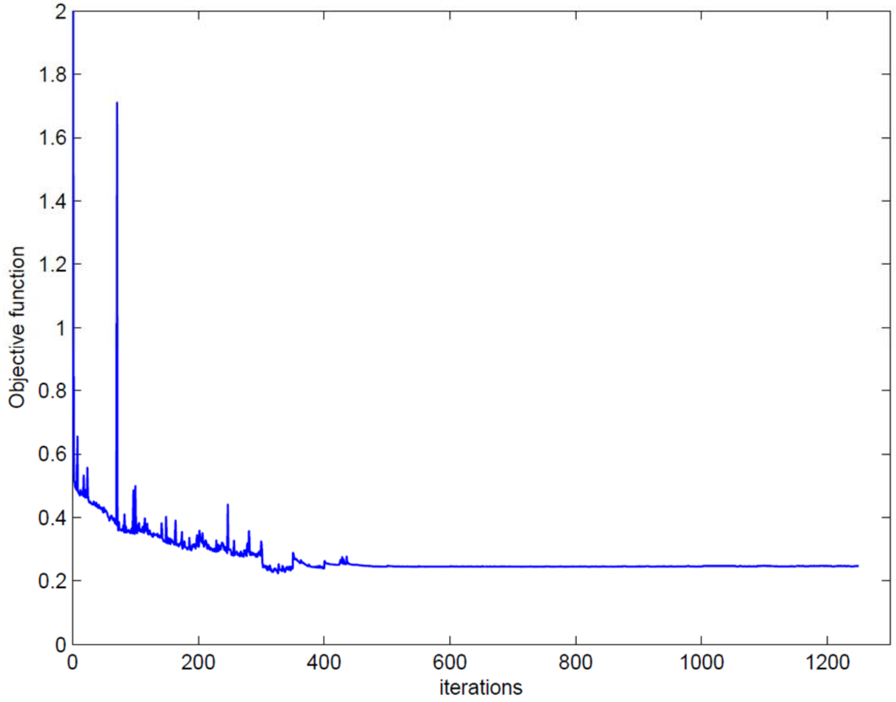
Final solution

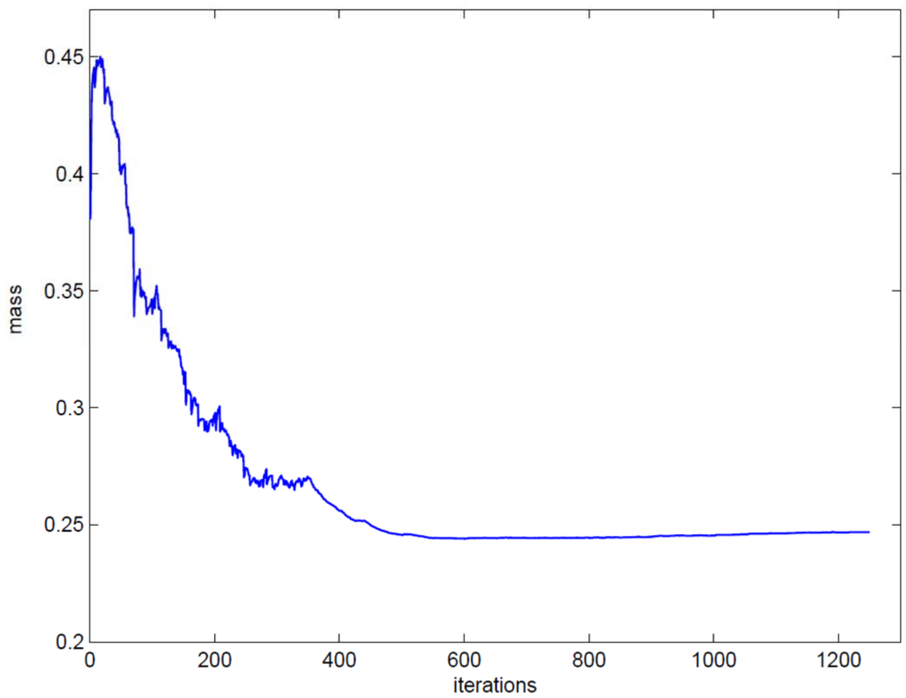
Numerical Tests

Failure function



Numerical Tests





Numerical Tests

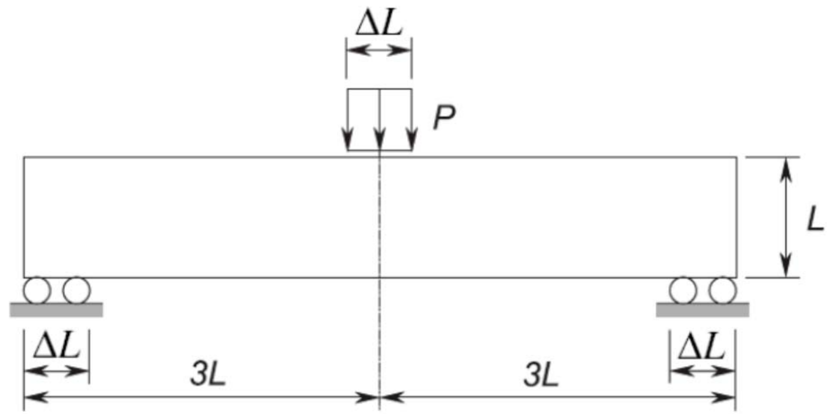
Example 4: MBB-beam

$$P = 2,0\ kN$$
$$\sigma_{adm} = 17,80\ kPa$$
$$L = 1,0\ m$$
$$\Delta L = 0,2\ m$$

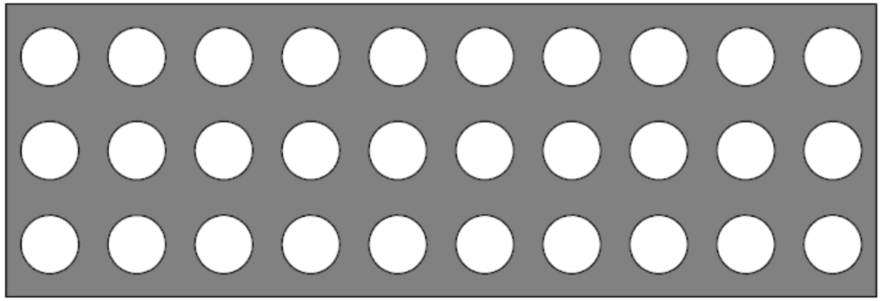
Elements mesh: 150 x 50

Grid level set: 151 x 51

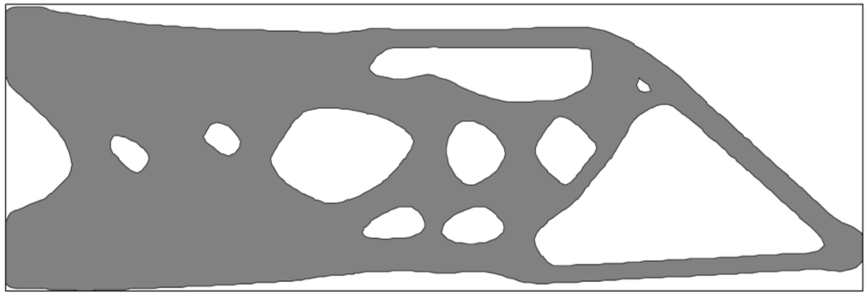
$$\xi = 0.7 \min(a,b)$$
$$\mu = 1e-7$$
$$c = 500$$



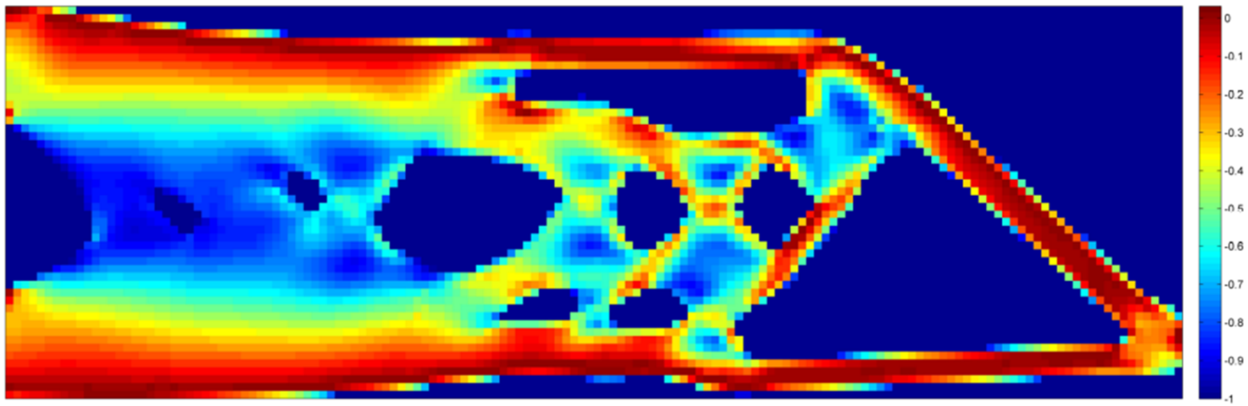
Numerical Tests



Initial design



Final solution



Failure function

Final Remarks

- ✓ Augmented Lagrangian approach provides a good representation of the constrained problem.
- ✓ Sensitivity analysis provides adequate directions for a minimization sequence.
- ✓ The approach "identifies" LOCAL high stress levels and modifies the shape according to that.
- ✓ Benchmarks solutions with optimum designs similar to those achieved in previous works.

Final Remarks

- The transport of Φ by the chosen solution of HJ do not keep *distance function* properties.
- Practical restarting techniques introduces shape changes grater than convergence conditions.
- Other stress failure criteria on "cut elements" must be tested.
- Success on minimization sequence still dependent on "good" parameters.

Thank you!

Obrigado!

Teşekkürler!