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An element-free Galerkin method approach for estimating sensitivity of machined surface parameters

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Abstract

The main objective of this study is to implement a parameter sensitivity analysis method to be used in the search of optimal machining conditions with respect to surface quality. Presently, the element-free Galerkin (EFGM) approximating functions are used to evaluate the properties of machined surfaces with cutting parameters when turning AISI 4140 steel using arbitrary sets of experimental values and the EFGM approximation functions, based on the moving least-squares method, in order to obtain the sensitivities through proper local derivations. This method shows the sensitivity of each surface parameter for each input variable. The variables investigated were cutting speed (v_c), depth of cut (a_p), feed rate (f) and the surface roughness (R_a). The sensitivity results showed that the feed rate has the highest influence on surface roughness when turning AISI 4140 steel followed by cutting speed and depth of cut.

Keywords: Parameter sensitivity; Element-free Galerkin method; Turning; Roughness

1. Introduction

Machining processes that remove chips from the work material by shearing action using a wedge-shaped tool, e.g. turning, milling, drilling, boring, etc., have been extensively studied in the past 50 years [1–4]. Machined surfaces can exhibit tensile or compressive residual stress, depending on the cutting parameters, cutting tool, tool wear, etc. On the other hand, the machined surfaces have some deviation characteristics that can be evaluated at micro- (or nano-) metric or at millimetre scale, denoted as roughness and form. The behaviour of machined components, under fatigue for instance, is highly influenced by the residual stress and surface roughness [5]. Therefore, knowing the effect of cutting parameters, like cutting speed (v_c), depth of cut (doc) and feed rate (f), on the machined surface is a great challenge for engineers and researchers. The method of factorial design has recently widely been intensively used to help in the planning of experiments in order to reduce the number of trials and also identify the individual influence of the parameters evaluated [6–8].

A number of meshless procedures have recently been proposed in the FEM community. These include the smoothed particle hydrodynamics method [9], the diffuse element method [10], wavelet Galerkin method [11], the element-free Galerkin method [12], the reproducing kernel particle method (RKPM) [13], the meshless local Petrov–Galerkin method [14], the natural element method [15], the partition of unity method [16] and the hp-cloud methods [17,18]. The latter has the further appeal of naturally introducing a procedure for performing hp-adaptivity in a very flexible way, avoiding the construction of functions by sophisticated hierarchical techniques. The cost can be reduced by using a linear Lagrangian partition of unity as in the finite element method as proposed by Oden et al. [19] and later denoted by the generalized finite element method (GFEM), which can be understood as a generalization of the partition of unity method. More recently, Belytschko

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and his co-workers proposed the extended finite element method (XFEM) [20], which presents similar characteristics as the GFEM.

Presently, using sets of experimental results of the element-free Galerkin method approximation functions, based on the moving least-squares method (LSM), are used for estimating surface quality sensitivities in terms of cutting parameters.

2. Parameter sensitivities

In order to approximate surface quality sensitivities with respect to a set of machining parameters using a set of experimental data will involve fitting a local polynomial function to this set using a weighted procedure, which is a generalization of the conventional LSM. To this end, consider a body occupying a domain $\Omega \in \Re^n$, $n = 1, 2$ or 3, with contour Γ, and let f_α, $\alpha = 0, 1, 2, 3, \ldots, N$, be the known values of a function $f(\mathbf{x})$ on an arbitrary set of N points $\mathbf{x}_\alpha \in \Omega$:

$$f_\alpha = f(\mathbf{x}_\alpha). \tag{1}$$

The main idea is to build an approximation function u_y of $f(\mathbf{x})$ at a point $\mathbf{y}$ in such a way that $u_y(\mathbf{x})$ essentially depends on the neighbouring values f_α. To this end, we define a functional $E_y(u_y)$ weighted by the functions $W_\alpha(\mathbf{y})$ in the following form:

$$E_y(u_y) = \frac{1}{2}\sum_{\alpha=0}^{n} W_\alpha(y)[u_y(x_\alpha) - f_\alpha]^2, \tag{2}$$

where $W_\alpha(\mathbf{y}) : \Re^n \to \Re$ are non negative functions with monotonic decreasing values with respect to the radius $\|\mathbf{y} - \mathbf{x}_\alpha\|$. These functions belong to the space W defined by

$$W = \{W_\alpha(y) \in C_0^s(\omega_\alpha), \quad s \geqslant 0 \ : \ W_\alpha(y) \geqslant 0, \quad \forall y \in \Re^n\}. \tag{3}$$

The supports ω_α of the weighting functions $W_\alpha(\mathbf{y})$ are open balls with radius h_α and centre in $\mathbf{x}_\alpha$, i.e.,

$$\omega_\alpha = \left\{\mathbf{x} \in \Re^n \ : \ \|\mathbf{x} - \mathbf{x}_\alpha\|_{\Re^n} \leqslant h_\alpha\right\}. \tag{4}$$

The approximation $u_y(\mathbf{x})$ is constructed, from many alternatives, as a linear combination of polynomials:

$$u_y(\mathbf{x}) = \sum_{j=0}^{m} a_j P_j(\mathbf{x}). \tag{5}$$

The $P_j(\mathbf{x})$ functions are components of a complete base $\mathbf{P}$ of polynomials of the order $m \leqslant N$:

$$\mathbf{P} = \left\{P_0(\mathbf{x}), P_1(\mathbf{x}), \ldots, P_j(\mathbf{x}), \ldots, P_m(\mathbf{x})\right\} \tag{6}$$

having the following property:

$$\mathbf{P} = \left\{P_i : \Re^n \to \Re, \quad P_i \in C^l, \ l \geqslant 0, \ i = 0, 1, \ldots, m\right\}. \tag{7}$$

The minimization of the functional given in Eq. (2) with respect to the generalized coordinates results in a system of $m+1$ equations, allowing the determination of the parameters a_i at point y such that

$$u_y(\mathbf{x}) = \sum_{j=0}^{m} a_j(\mathbf{y}) P_j(\mathbf{x}). \tag{8}$$

This procedure can be understood as a local approximation $u_y(\mathbf{x})$ of function f at point $\mathbf{y}$. In the traditional LSM, the weight $W_\alpha(\mathbf{y})$ is constant while the coordinate a_j depends on the local point $\mathbf{y}$ where the function u_y is presently being approximated.

In order to clarify the procedure, it is convenient to define the following arrays:

$$\mathbf{V} = \begin{bmatrix} P_0(\mathbf{x}_0) & P_1(\mathbf{x}_0) & \ldots & P_m(\mathbf{x}_0) \\ P_0(\mathbf{x}_1) & P_1(\mathbf{x}_1) & \ldots & P_m(\mathbf{x}_1) \\ \ldots & \ldots & \ldots & \ldots \\ P_0(\mathbf{x}_N) & P_1(\mathbf{x}_N) & \ldots & P_m(\mathbf{x}_N) \end{bmatrix}, \tag{9}$$

$$\tilde{\mathbf{W}}(\mathbf{y}) = \begin{bmatrix} W_0(\mathbf{y}) & 0 & \ldots & 0 \\ 0 & W_1(\mathbf{y}) & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & W_N(\mathbf{y}) \end{bmatrix}, \tag{10}$$

$$\mathbf{a}(\mathbf{y}) = \begin{bmatrix} a_0(\mathbf{y}) \\ a_1(\mathbf{y}) \\ \ldots \\ a_m(\mathbf{y}) \end{bmatrix} \tag{11}$$

and

$$\mathbf{f}(\mathbf{y}) = \begin{bmatrix} f_0(\mathbf{y}) \\ f_1(\mathbf{y}) \\ \ldots \\ f_m(\mathbf{y}) \end{bmatrix}. \tag{12}$$

With these notations, the local approximation $u_y(\mathbf{x})$ and the functional $E_y(u_y)$ can be rewritten as

$$u_y(\mathbf{x}) = \mathbf{V}\,\mathbf{a}(\mathbf{y}), \tag{13}$$

$$E_y(u_y) = \frac{1}{2}\tilde{\mathbf{W}}(\mathbf{y})(\mathbf{V}\,\mathbf{a}(\mathbf{y}) - \mathbf{f})(\mathbf{V}\,\mathbf{a}(\mathbf{y}) - \mathbf{f}). \tag{14}$$

The minimum value of these functionals must be achieved by a function satisfying the following first-order condition:

$$DE_y(u_y) = \tilde{\mathbf{W}}(\mathbf{y})(\mathbf{V}\,\mathbf{a}(\mathbf{y}) - \mathbf{f}) \cdot \mathbf{V}\,\hat{\mathbf{a}} = \mathbf{0}, \quad \forall \hat{\mathbf{a}} \in \Re^n. \tag{15}$$

Thus, as $\hat{\mathbf{a}}$ is an arbitrary vector, the following system of equations is satisfied:

$$\mathbf{V}^{\mathrm{T}}\tilde{\mathbf{W}}(\mathbf{y})\mathbf{V}\,\mathbf{a}(\mathbf{y}) = \mathbf{V}^{\mathrm{T}}\tilde{W}(\mathbf{y})\mathbf{f}. \tag{16}$$

The solution of this system supplies the vector of parameters $\mathbf{a}$. This can be written in a more compact form as

$$\mathbf{A}(\mathbf{y})\mathbf{a}(\mathbf{y}) = \mathbf{F}(\mathbf{y}), \tag{17}$$

where

$$\mathbf{A}(\mathbf{y}) = \mathbf{V}^{\mathrm{T}}\tilde{\mathbf{W}}(\mathbf{y})\mathbf{V} \tag{18}$$

and

$$\mathbf{F}(\mathbf{y}) = \mathbf{V}^{\mathrm{T}}\tilde{\mathbf{W}}(\mathbf{y})\mathbf{f}. \tag{19}$$

The matrix $\mathbf{A}(\mathbf{y})$ must satisfy a set of minimum properties to guarantee the existence of its inverse [10].

An approximation of the sensitivity of an arbitrary function $f(\mathbf{x})$ at a point $\mathbf{x} = \mathbf{y}$ can be obtained by evaluating the derivates of Eq. (8). Basically,

$$u(x) = \mathbf{p}^{\mathrm{T}}(x)\,\boldsymbol{\alpha}(x), \tag{20}$$

therefore its derivate is given by

$$u_{,s}(x) = \mathbf{p}_{,s}^{\mathrm{T}}(x)\boldsymbol{\alpha}(x) + \mathbf{p}^{\mathrm{T}}(x)\alpha_{,s}(x). \tag{21}$$

Similarly, higher-order sensitivities can be obtained in a straightforward manner. The continuity of the approximating function is the same as the weighting function. Presently, the function $W_\alpha(y) = 4/\pi[1-(y-x_\alpha/2h_\alpha)^2]^4$ [9] is used for estimating the sensitivities of surface roughness (R_a) with respect to the cutting parameters comprising cutting speed (v_c), depth of cut (doc) and feed rate (f).

3. Experimental procedures

A CNC machine tool with a 10 kW motor drive and a maximum spindle rotation of 4000 rpm was used for machining. The work material is AISI 4140 steel bar with length 300 mm, diameter 100 mm and hardness 190 HV. The tool material is cemented carbide coated with TiN using physical vapour deposition (PVD) technique. The turning tests were carried out without coolant at the following conditions:

Cutting speed, v_c: 50, 150 and 300 m/min,
Feed rate, f: 0.03, 0.1 and 0.3 mm/rev and
Depth of cut, doc: 0.5, 1.5 and 3 mm.

Each test was carried out with a new cutting edge and for a maximum cutting length of 10 mm. The surface finish generated was recorded and the results were used in the sensitivity method developed in this study.

In order to process the mathematics of the parameter sensitivity analysis, a software was written using the Matlab® program.

4. Results and discussions

Fig. 1 shows plots of the fitted variation of surface roughness (R_a) with cutting speed using experimental data as input for the computational sensitivity method while the evaluated surface roughness sensitivity with cutting speed is shown in Fig. 2. Increase in cutting speed has a tendency of improving the surface finish and thus reducing the R_a parameter [3,21]. This is attributed to increased heat

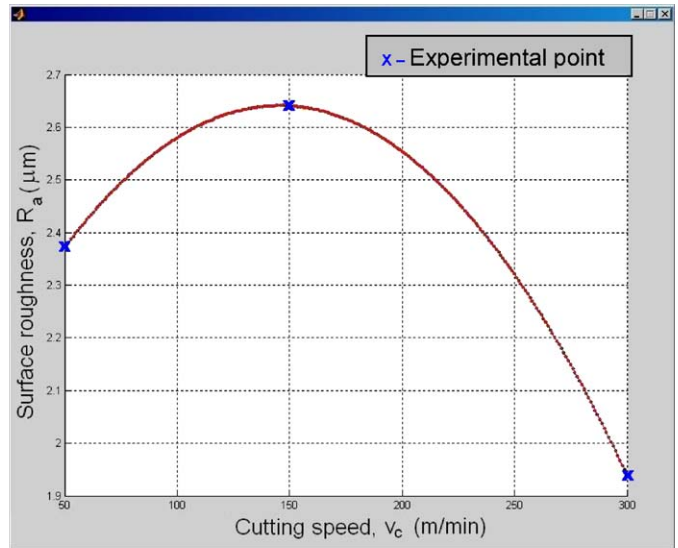

Fig. 1. Fitted data of surface roughness (R_a) with the cutting speed (v_c).

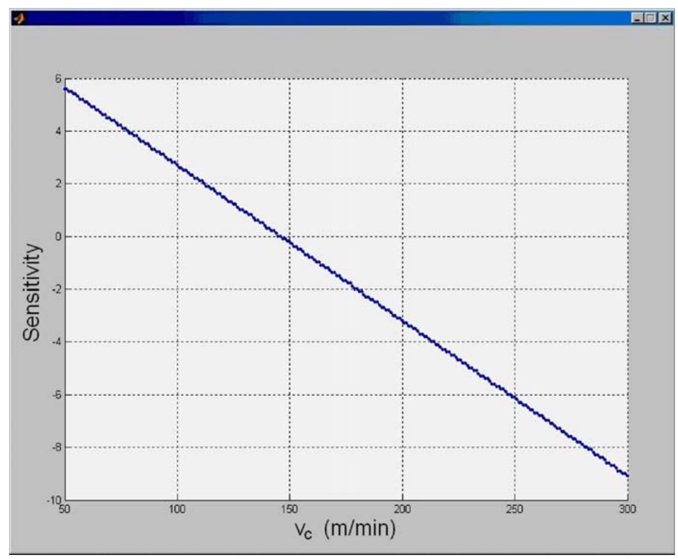

Fig. 2. Variation of surface roughness (R_a) sensitivity with cutting speed (v_c).

generation at the chip–tool and tool–work piece interfaces during machining. The associated temperature increase generally lowers the cutting forces, consequently smothering the cutting processes. This phenomenon occurs at cutting speeds in excess of 150 m/min when machining steels [1]. It can be observed in Fig. 2 that the R_a sensitivity reduces with increasing cutting speed. Positive R_a sensitivity value means there is proportionality between R_a and cutting speed. The surface roughness range recorded during the turning process is between 1.94 and 2.38 µm.

Trent and Wright [1] reported the occurrence of built-up edge (BUE) when machining multiphase materials at lower cutting speeds. BUE formation is significantly reduced when the cutting speed is increased. On the other hand, the presence of BUE generates large burr quantity on the machined surface, consequently deteriorating surface finish. Fig. 1 shows that the R_a values increased with cutting speed from 50 to 150 m/min. The R_a values decreased when machining at higher speeds in excess of 150 m/min. This behaviour is reinforced in Fig. 2. It can be observed that the sensitivity value is positive at lower cutting speeds below 150 m/min. The sensitivity value reduces with increasing cutting speed and becomes negative at cutting speeds above 150 m/min. It can also be observed that the magnitudes of sensitivities are higher at negative values when compared with the positive values. This means that machining at cutting speed above 150 m/min have more influence on surface roughness value.

Fig. 3 shows a plot of the fitted variation of surface roughness (R_a) with the depth of cut while the calculated surface roughness sensitivity curve is shown in Fig. 4. Increase of the depth of cut worsens the surface finish generated as illustrated in Fig. 3. This phenomenon may be associated with increase in the cutting forces and the consequent dynamic instability of the cutting process. It can be seen in Fig. 4 that positive sensitivity was recorded when machining at a depth of cut up to 2.75 mm. Beyond this value the sensitivity becomes negative. In other words, increase in the depth of cut increases surface roughness values. The sensitivity value is zero close to a depth of cut of 2.75 mm.

Fig. 5 shows plots of the fitted variation of surface roughness (R_a) with the feed rate while the calculated surface roughness sensitivity is illustrated in Fig. 6. Increases in the feed rate from 0.03 to 0.17 mm/rev have the tendency to increase the surface roughness value (R_a)

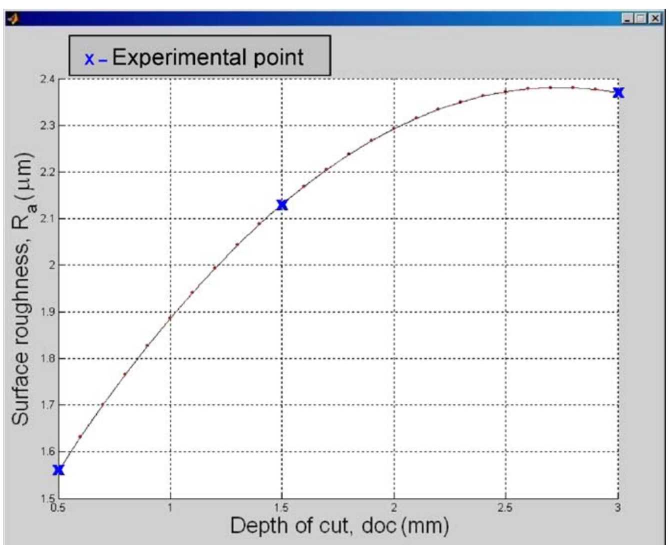

Fig. 3. Fitted data of surface roughness (R_a) with the depth of cut (doc).

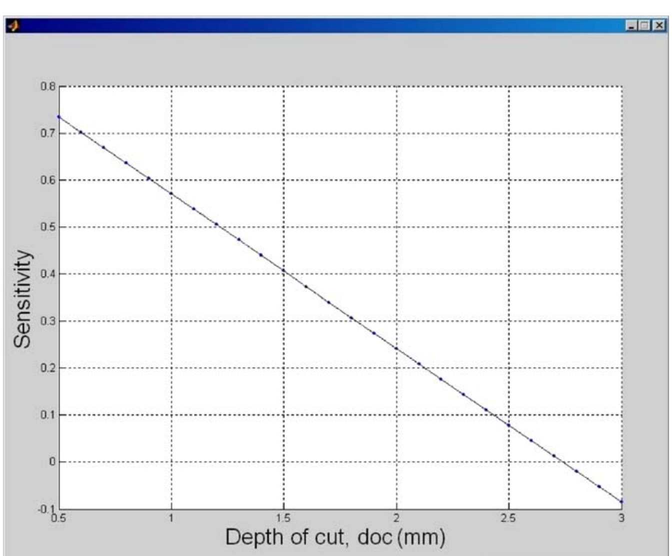

Fig. 4. Variation of surface roughness (R_a) sensitivity with the depth of cut (doc).

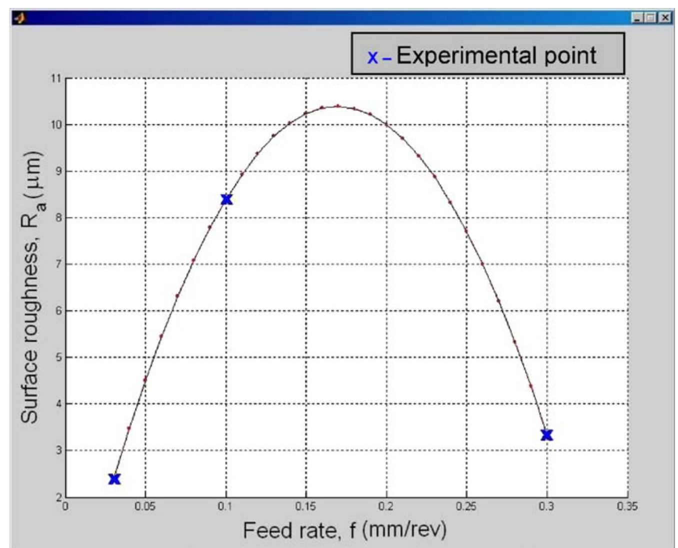

Fig. 5. Fitted data of surface roughness (R_a) with feed rate (f).

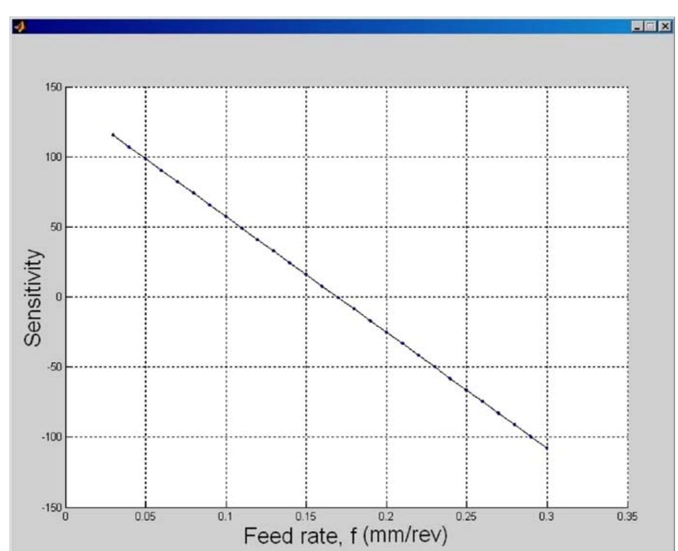

Fig. 6. Variation of surface roughness (R_a) sensitivity with feed rate (f).

while a reverse trend was observed when machining a higher feed rate from 0.17 to 0.3 mm/rev. In general, there is a direct relation between feed rate and surface roughness when machining with machine tools that exhibit a high dynamic stability. In this particular case, undesirable vibrations were recorded when machining at feed rates between 0.1 and 0.2 mm/rev. This must be responsible for the high surface roughness values recorded under these conditions. Higher feed rates, in excess of 0.2 mm/rev, showed significant improvement in the dynamic stability of the system in addition to the low vibration recorded.

Machining at feed rates from 0.03 to 0.17 mm/rev produced positive sensitivity (Fig. 6). The sensitivity became negative when machining at feed rates above 0.17 mm/rev.

The magnitude of sensitivity values provides a good indication of the influence of machining parameters on surface roughness. It can be seen in Fig. 2 that the cutting speed promotes the maximum absolute value of R_a sensitivity of 9 and the range recorded was from -9 to 5.4. In Fig. 4, the depth of cut promoted a maximum absolute value of R_a sensitivity of 0.74 and the range recorded was 0.8–0.74. Finally, the feed rate promotes the maximum absolute value of R_a sensitivity of 130 and the recorded values ranged from -110 to 130. It is therefore evident from this study, using the proposed method, that the feed rate is the most important cutting parameter that influences the machined surface followed by the cutting speed and finally the cut depth when machining AISI 4140 steel.

One can note from Figs. 2, 4 and 6 that the sensitivity varies linearly even though higher-order polynomials were used. This confirms that the most relevant parameter, R_a, in this experiment is given by

$$R_a = \frac{f^2}{31.2 r_n},$$

where r_n is the tool tip radius which is constant in this case.

Therefore, results from such figures indicate not only the influence of the cutting parameters on the surface quality, but also ensure optimization of the operating conditions once preliminary sets of experiments are performed. This can be noted in Fig. 6 where the effect of feed rate on surface roughness, R_a, values reverses for feed rates higher than 0.17 mm/rev.

The proposed scheme is generic and may be used in other manufacturing processes. Since machining is a highly nonlinear phenomenon, each process will require a new set of exploratory experiments for evaluation of the sensitivities and optimization of the operational conditions.

5. Conclusions

- The sensitivity method proposed in this study is a powerful computational tool to help analysis of the correlation between the cutting parameters and the surface roughness of machined surfaces without embarking on laborious time consuming and often expensive machining trials.
- The sensitivity results showed that the feed rate is the most important cutting parameter for determining the machined surface roughness, R_a, when turning AISI 4140 steel. This is followed by the cutting speed and the depth of cut in ranking order.

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