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APPENDIX

For a (x, y) Cartesian system centered at the singularity pole $\xi = 0$, one may always write

$$x = \xi\bar{x}, \quad y = \xi\bar{y}, \quad r = (x^2 + y^2)^{1/2} = \xi\bar{r} \tag{A.1}$$

where $\bar{x} \equiv \bar{x}(\xi)$, $\bar{y} \equiv \bar{y}(\xi)$ and $\bar{r} \equiv \bar{r}(\xi)$ are different from zero along the whole integration interval. Then, it may be verified that

$$\frac{dr}{d\xi} = \bar{r} + \frac{\xi}{\bar{r}}\left(\bar{x}\frac{d\bar{x}}{d\xi} + \bar{y}\frac{d\bar{y}}{d\xi}\right) \tag{A.2}$$

Moreover, the Jacobian of the transformation between the Cartesian coordinates (x, y) and the natural coordinate ξ may be expressed as

$$|J| = \left(\left(\frac{dx}{d\xi}\right)^2 + \left(\frac{dy}{d\xi}\right)^2\right)^{1/2} = \left(\left(\xi\frac{d\bar{x}}{d\xi} + \bar{x}\right)^2 + \left(\xi\frac{d\bar{y}}{d\xi} + \bar{y}\right)^2\right)^{1/2} \tag{A.3}$$

From eqs. (A.2) and (A.3), one may express for the singularity pole $\xi = 0$:

$$|J_0| = \frac{dr_0}{d\xi} = \bar{r}_0 \tag{A.4}$$

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GREEN'S ELEMENT METHOD - PART I: *ASSEMBLED LOCAL FORM*

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Abstract

The Modified Local Green's Function Method, MLGFM, has been proposed as an alternative for extending the applicability of the Boundary Element Method to problems which do not have a known fundamental solution. Up to now, most of the published results are based on a global version where only one Green's Cell is used. In this paper, a scheme for solving multicell models is presented and some results are compared against Mixed Finite Element solutions. This comparison illustrates the beneficial effects that the boundary elements bring in the formulation: improved accuracy and fewer iterations in nonlinear problems.

1-Introduction

Although the Boundary Element Method, BEM, has long been recognized as an established numerical method with many advantages over other techniques, its formulation may become cumbersome when a fundamental solution is not available or has a too complicated form. The Modified Local Green's Function Method, MLGFM, from now on also called Green's Element Method, GEM, was proposed by de Barcellos & Silva [1] as an alternative to extend the BEM methodologies to problems whose fundamental solution in its explicit form is either unknown or very intricate. Some examples of these are potential problems in non-homogeneous media, deformation of plates with variable thickness and shells of arbitrary shape.

Most of the already published work, see e.g. [2, 3, 4, 5], on the MLGFM have made use of the global version, just one Green's cell, where the accuracy and convergence properties have been investigated. Essentially, the MLGFM uses a mesh of Green's cells where each one of them requires a coarse finite element, FE, mesh and associated boundary element, BE, mesh in order to locally

evaluate the Green's function projections on the FE and BE interpolations functions. After these projections are computed, the Galerkin-BEM formalism is applied. The boundary values and continuity requirements between the cells are introduced as in BEM and the boundary cell unknowns are evaluated. If the values of the unknown functions inside a cell are desired, one may again,use the MLGFM in such a cell, now designated as *macrocells*, by using its solution as boundary conditions for the *microcell*' s mesh into which it is divided. In doing so, one is using the same sort of ideas as domain partition and multi-grid concepts.

2 - Modified Local Green's Function

Consider the differential equation for a non-homogenous potential problem

$$\begin{aligned} -\nabla \cdot Z(u,P)\nabla u &= b(P), && P\in\Omega \\ u(P) &= \bar{u}(p), && p\in\Gamma_u \\ -\hat{n}\cdot Z\nabla u &= f(p) = \bar{f}(p), && p\in\Gamma_f \end{aligned} \tag{1}$$

where: Ω, Γ_u and Γ_f stand for the domain and boundary partitions where the Dirichlet, $\bar{u}$,and Neumann, $\bar{f}$, boundary conditions are specified; $Z(u,P)$ is a material property which might be also function of the potential u; and b is the excitation function. Initially, the domain is partitioned in a coarse mesh of Green's cells. Next, consider for each cell the integral equation

$$\begin{aligned} u(Q) = &\int_{\Omega_P} G(P,Q)b(P)\,d\Omega + \oint_{\Gamma_P}[u_{,n}+\alpha(p)u(p)]G(p,Q)d\Gamma \\ &-\oint_{\Gamma_P}[G_{,n}+\alpha(p)G(p,Q)]u\,d\Gamma \end{aligned} \tag{2}$$

where α is a convenient scalar operator and G is the Green's function of the First Auxiliary Problem:

$$\begin{aligned} -\nabla_P^2 G(P,Q) &= \delta(P,Q) && \forall P,Q\in\Omega \\ G_{,n}+\alpha(p)G(p,Q) &= 0 && \forall p\in\Gamma \end{aligned} \tag{3}$$

Defining $F(p) = u_{,n}+\alpha(p)u(p)$ and considering (3), one can rewrite (2) as

$$u(Q) = \int_{\Omega_P} G(P,Q)b(P)\,d\Omega + \oint_{\Gamma_P} F(p)G(p,Q)\,d\Gamma,\ \forall Q\in\Omega,\ p\in\Gamma \tag{4}$$

Define the Second Auxiliary Problem

$$\begin{aligned} -\nabla_P^2 G(P,q) &= 0 && \forall P\in\Omega, q\in\Gamma \\ G_{,n}+\alpha(p)G(p,q) &= \delta(p,q) && \forall p,q\in\Gamma \end{aligned} \tag{5}$$

Using (5) and taking the limit in (2) as the inner point "Q" approaches a boundary point "q", one obtains

$$u(q) = \int_{\Omega_P} G(P,q)b(P)d\Omega + \oint_{\Gamma_P} F(p,q)G(p,q)d\Gamma\ ,\forall P\in\Omega,\ p,\ q\in\Gamma \tag{6}$$

One can notice that equations (2) and (6) are the same equations used in BEM except for the variable α and that G now stands for the Green's function associated with the two auxiliary problems. The next step is to use the Galerkin Method on the equations (4) and (6). For this purpose, one defines a set of FE interpolations, $\{\psi_i\}$, defined on each domain partition (cell), and a set of BE interpolation

functions, $\{\phi_i\}$, such that each ϕ_j is the trace of a ψ_k. As a result of the Galerkin procedure, one obtains two sets of algebraic equations, namely

$$Au^d = BF^c + Cb^d \qquad Du^c = EF^c + Fb^d \tag{7}$$

where u^d and u^c are the nodal domain and boundary values of u, respectively, and the matrices are defined as

$$\begin{aligned} A &= \int_{\Omega_Q}[\psi(Q)]'[\psi(Q)]d\Omega_Q & D &= \oint_{\Gamma_q}[\phi(q)]'[\phi(q)]d\Gamma_q \\ B &= \oint_{\Gamma_P} G_d(p)'[\phi(p)]d\Gamma_P & E &= \oint_{\Gamma_P} G_c(p)'[\phi(p)]d\Gamma_P \\ C &= \int_{\Omega_P} G_d(P)'[\psi(P)]d\Omega_P & F &= \int_{\Omega_P} G_c(P)'[\psi(P)]d\Omega_P \\ G_d(p)' &= \int_{\Omega_Q}[\psi(Q)]' G(p,Q)'d\Omega_Q & G_c(p)' &= \oint_{\Gamma_q}[\phi(q)]' G(p,q)'\,d\Gamma_q \\ G_d(P)' &= \int_{\Omega_Q}[\psi(Q)]' G(P,Q)'d\Omega_Q & G_c(P)' &= \oint_{\Gamma_q}[\phi(q)]' G(P,q)'\,d\Gamma_q \end{aligned} \tag{8}$$

The Green's function projections, G_d and G_c are obtained by projecting the quantities in equations (3) and (5) over the subspaces generated by $\{\psi_i\}$ and $\{\phi_i\}$, respectively, then:

$$\begin{aligned} -\nabla_P^2 G_d(P) &= [\psi(P)], && \forall P\in\Omega \\ G_{d,n}(p)+\alpha(p)G_d(p) &= 0, && \forall p\in\Gamma \end{aligned} \tag{9}$$

$$\begin{aligned} -\nabla_P^2 G_c(P) &= 0, && \forall P\in\Omega \\ G_{c,n}(p)+\alpha(p)G_c(p) &= [\phi(p)], && \forall p\in\Gamma \end{aligned} \tag{10}$$

Applying the Finite Element Method, FEM, to solve (9) and (10) one obtains [2] :

$$[K_o + K_2][G^{DP};G^{CP}] = [M;m] \tag{11}$$

where: M is the mass matrix with unit density; m is modified mass matrix defined on the boundary; K_o is the usual FEM "stiffness" matrix and K_2 is a matrix which comes from a boundary integral involving the α parameter, weighting and interpolation functions [7]; G^{DP} and G^{CP} are the Green's functions projection coefficients. Using the definitions (8) and (11), the matrices in (7) become

$$B = G^{DpJ}D, \qquad F = G^{CPJ}A, \qquad E = G^{CpJ}D \tag{12}$$

It can be shown that [7], for self adjoint problems,

$$\begin{aligned} B &= AG^{CP} & E &= DG^{Cp} \\ C &= AG^{DP} & F &= DG^{Dp} \end{aligned} \tag{13}$$

3 - Local Form of the MLGFM

The solution of the elliptic boundary value problem by the MLGFM is obtained by the simultaneous solution of the algebraic systems of equations (7). The matrices B, C and E, F can be written in terms of A and D as shown in (13). From the definition of A and D, Eqs. (8), we see that they are symmetric and nonsingular, therefore they can be dropped from Eq. (7), resulting in the systems:

$$\begin{aligned} u^d &= G^{CP}F^c + G^{DP}b^d \\ u^c &= G^{Cp}F^c + G^{Dp}b^d \end{aligned} \tag{14}$$

It is observed that the problems (11) are defined by an unique set of boundary conditions and loading. This auxiliary problem depends on the material and geometrical parameters of the *real problem* which is to be solved, but not on the its loading or boundary conditions. This can be seen by observing that the auxiliary problems are defined from the adjoint domain and boundary operators to the original operators, and are defined for the same domain and boundary, and for the same material properties of the original problem; but, as seen on the right hand side of Eqs. (11), the excitations on the domain and the specified values on the boundaries are not those of the physical problem. They will be always those indicated in Eqs. (9) and (10). This makes it possible to define Eqs. (15) and (14) , but for only part of whole domain, a *subdomain* or *cell* . For example, Figure 1 shows the case where the domain is partitioned into the two cells 1 and 2.

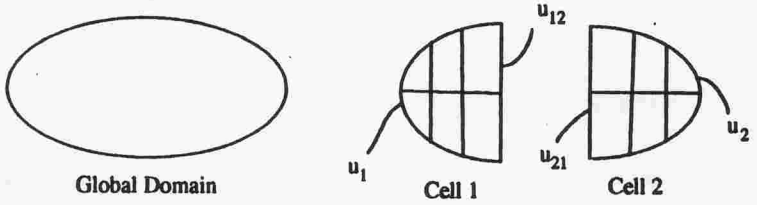

Figure 1. Domain Partition into Green's Cells.

We divide the boundary of subdomain 1 in a part 1 which is common with the boundary of the global domain and a part 12 at the interface between both subdomains. Similarly we define parts 2 and 21 for the subdomain 2. The nodal unknowns u and F have their nomenclature changed accordingly, as indicated in Figure 1 for the primary variables u. Now one may rewrite the expression (14.b) for the macrocell "i" only, still using the notation of (7), as if it were an independent problem:

$$\begin{bmatrix} D_i & D_{ij} \end{bmatrix} \begin{bmatrix} u_i^c & u_{ij}^c \end{bmatrix}^t - \begin{bmatrix} E_i & E_{ij} \end{bmatrix} \begin{bmatrix} F_i^c & F_{ij}^c \end{bmatrix} + \begin{bmatrix} F_i & F_{ij} \end{bmatrix} \begin{bmatrix} b_i^d & b_{ij}^d \end{bmatrix} \tag{16}$$

where the superscript t means transpose of the array. The indices 1, 2, etc., indicate the submatrices formed by collecting the columns which correspond to the degrees of freedom of the part of the boundary referred in the indices. The coefficient matrices D, E and F are defined analogously to the coefficient matrices shown in Eqs. (14). These coefficients are obtained by solving systems of the type shown in Eqs. (11), but now all the matrices are defined over the corresponding cell only.

Equation (16) can be combined to furnish the solution on the boundary, but clearly there is an excess of unknowns. It is necessary to impose *conditions of compatibility* between the nodes at the interface. For primitive variables, such as displacements or velocities, one requires that $u_{12}^c = u_{21}^c$. One must remember that the fluxes F appearing so far, are *apparent fluxes* related the real (physical) fluxes f by: $F_{12} = f_{12} + N_{12}^{'} u_{12}$ and $F_{21} = f_{21} + N_{21}^{'} u_{21}$.

The normal vectors at the interfaces of the two neighbor cells have opposite directions. Therefore the real fluxes obey $f_{12} = -f_{21}$. In this way the system of Equations (16) become

$$\begin{bmatrix} (D_1 - N_1 E_1) & (D_{12} - N_{12} E_{12}) & 0 \\ 0 & (D_{21} - N_{21} E_{21}) & (D_2 - N_2 E_2) \end{bmatrix} \begin{bmatrix} u_1 \\ u_{12} \\ u_2 \end{bmatrix} =$$

$$\begin{bmatrix} E_1 & E_{12} & 0 \\ 0 & -E_{21} & E_2 \end{bmatrix} \begin{bmatrix} f_1^c \\ f_{12}^c \\ f_2^c \end{bmatrix} + \begin{bmatrix} F_1 & F_{12} & 0 \\ 0 & F_{21} & F_2 \end{bmatrix} \begin{bmatrix} b_1^d \\ b_{12}^d \\ b_2^d \end{bmatrix}, \tag{17}$$

or, to short notation, we may define the barred matrices such that:

$$\overline{D} u^c = \overline{E} f^c + \overline{F} b^d . \tag{18}$$

We note that Eqn. (18) is written in terms of the nodal real fluxes f^c instead of the apparent ones F^c.

4 - FAST WAYS TO OBTAIN THE MATRICES E AND F.

For each cell, the matrices E and F shown in Eqs. (16) and (17) can be rewritten using (13), eliminating D and leaving only partitions of G^{Dp} and G^{Cp}. These matrices are formed by collecting the rows of G^{DP} and G^{CP} respectively, which correspond to the degrees of freedom (dof's) on the boundary of the cell considered. This implies that the dof's belonging to the double nodes at the corners, are defined in G^{Dp} and G^{Cp} by simply duplicating the corresponding rows of G^{DP} and G^{CP}.

Each cell can be partitioned by an arbitrary number of finite elements in its domain, and by a mesh of boundary elements whose basis functions are the limit of the finite element basis functions as they approach the boundary. It is considered here the special case where each cell is composed by one single finite element. Take for instance the Eqn. (11) for G^{DP}. One can reorder the rows and columns of K, G^{DP} and M grouping then in: d_i dof's associated with the internal nodes; d_s dof's associated with the simple B.E. nodes; and d_d dof's associated with the double B.E. nodes. These arrays are then partitioned as follow

$$K_a = \begin{bmatrix} K_{ii} & K_{is} & K_{id} \\ K_{si} & K_{ss} & K_{sd} \\ K_{di} & K_{ds} & K_{dd} \end{bmatrix} \quad G_a = \begin{bmatrix} G_i \\ G_s \\ G_d \end{bmatrix} \quad M_a = \begin{bmatrix} M_i \\ M_s \\ M_d \end{bmatrix}, \tag{19}$$

where the subscripts i, s and d refer to degrees of freedom associated with internal, single and double nodes respectively. In this form M_a is no longer symmetric since only the rows were reordered. If necessary, the simple reordering of the columns in G^{DP} and M_a will recover the symmetry of M.

Next we create a new matrix G_b by taking G_a and doubling the rows associated with the double nodes, the submatrix G_d, i.e.

$$G_b = \begin{bmatrix} G_i & G_s & G_d & G_d \end{bmatrix}^t . \tag{20}$$

Compatible with G_b, we define new arrays K_b and M_b such that the satisfaction of the relation (15) will imply that

$$K_b G_b = M_b . \tag{21}$$

With regard to the second part of Eq. (11), we similarly define g_b, K_b and m_b such that

$$K_b\, g_b = m_b. \tag{22}$$

Next we rewrite (21) in the following partitioned form

$$\begin{bmatrix} K_{ii} & K_{ie} \\ K_{ei} & K_{ee} \end{bmatrix} \begin{bmatrix} G_i \\ G_e \end{bmatrix} = \begin{bmatrix} M_i \\ M_e \end{bmatrix}, \tag{23}$$

where the subscript e indicates external nodes. The Auxiliary Problem (22) on the boundary can also be partitioned to

$$\begin{bmatrix} K_{ii} & K_{ie} \\ K_{ei} & K_{ee} \end{bmatrix} \begin{bmatrix} g_i \\ g_e \end{bmatrix} = \begin{bmatrix} m_i \\ m_e \end{bmatrix}, \tag{24}$$

where g_e, m_e and G_e are defined similarly to M_e. Static condensation to problems (23) and (24) results in

$$\overline{K}\,[G_e\ g_e] = [\overline{M}\ \overline{m}] \quad \text{with} \quad \begin{cases} \overline{K} = K_{ee} - K_{ei} K_{ii}^{-1} K_{ie} \\ \overline{M} = M_e - K_{ei} K_{ii}^{-1} M_i \\ \overline{m} = m_e - K_{ei} K_{ii}^{-1} m_i. \end{cases} \tag{25}$$

In case the basis and trial functions are chosen such that there is partial or complete uncoupling between internal and external degrees of freedom, i.e., $K_{ei} = 0$ and/or $K_{ie} = 0$, the right hand sides of Eqs (23.10) become independent of G_e and g_e, and $\overline{K} = K_{ee}$. In case $K_{ei} = 0$ or $K_{ii} = 0$, it is not necessary to compute K_{ii}^{-1}, and $\overline{M} = M_e$ and $\overline{m} = m_e$. We notice that G_e and g_e are precisely the coefficients of the projections on the boundary, the matrices G^{Dp} and G^{Cp} appearing in Eqn. (14). Now we can premultiply Eqn. (14b) by $\overline{K}$ and use Eqs. (25) to find

$$\overline{K}\, u^c = \overline{m} f^c + \overline{M} b^d. \tag{26}$$

This expression brings up an important point. *Here, in the case of the semi-orthogonality described above, we have obtained the equation for the nodal unknowns on the boundary of the macrocell without having to solve previously any algebraic system of equations., but simply performing row/column permutations. It is no longer necessary to determine explicitly the Green's Functions Projections from Eqn. (11) at each cell. The only steps remaining consist in computing K_{ee}, M_e and m_e for each macrocell and assembling them for the whole model in a form similar to that shown in Eqn. (17) for two cells.*

We notice that, even though the Green's Function Projections were not determined explicitly, their effect is still present in the formulation through the coefficient matrices in (26). The semi-orthogonality required above occur very naturally when the cell is partitioned by one single element defined by basis and trial functions of the Serendipity family. Even in the absence of any uncoupling,

the above formulation can be useful. Consider the classical Lagrangian element of degree p. The total number of nodes in the element is $n_{noe} = (p+1)^2$ and the number of internal nodes is $n_{ni} = (p-1)^2$.

5 - DETERMINATION OF THE STRESSES.

Consider first the case in which the Green's elements used are the four-noded bilinear quadrilateral (considering two-dimensional analysis for simplicity). The solution of the assembled system (18) will give the tractions at each of the B.E. nodes. Since all nodes are double, the solution will be known in two directions for each corner of the element, as indicated in Figure 2 for the simpler case of the isotropic Poison's problem. In this case we have computed the fluxes f at all 8 boundary nodes of each cell. For a given double boundary node like 4-5 of the Figure 2, the fluxes at both nodes are sufficient to compute the derivatives $u_{,x}$ and $u_{,y}$ by the definition of the fluxes: $f(x,y) = -\nabla u \cdot n$, where n is the outward unit normal vector to the point. For other types of problems, linear plane elasticity for instance, the computation is analogous: the tractions determined in two different surfaces about the same point are sufficient to compute all stress tensor components at that point. Once the fluxes (or stress tensor) is computed at the four corners, each component of stress can be interpolated to the internal points of the element using the bilinear basis functions.

Now consider an higher-order Green's Element. Figure 2.b shows a biquadratic element used for two-dimensional linear elasticity. After the first step of solution we have all tractions on the boundary of the cell. To simplify the notation in this example, the element is rectangular with faces normal to the coordinate axis such that the tractions coincide with the stress components, as depicted in two of the nodes in the Figure. At the four nodes on the corners we already know the state of stress. At external single nodes like node 6, on the other hand, the knowledge of two of the tractions is not sufficient to compute all three stress components at the point. The tractions at the four corners are still sufficient to express individually each component in a bilinear distribution over the interior of the cell, resulting in a coarse estimate of the pointwise stresses.

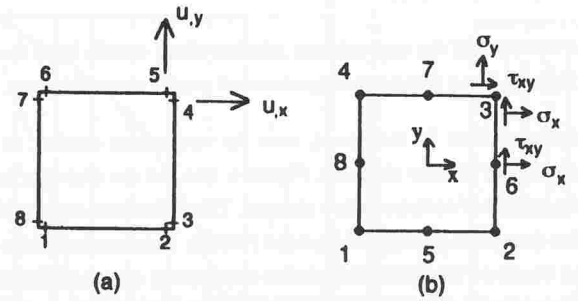

Figure 2. a) Bilinear and b) biquadratic Green's element.

In regions where a more precise distribution of the stresses is required, post-processing can be done at designated cells. After the principal analysis has been done, the displacements and tractions are known at the boundary of every cell. Each of these selected high-order cells, also referred as macro-cells or Green's Elements, can now be subdivided into new 4-noded Green's sub-cells. An analysis

similar to that of Eqs. (18) can be performed, this time only at the level of the macro-cell, using the just computed values of displacement and tractions as boundary values applied on the reduced problem.

6 - APPLICATION

Find the approximate solution for the problem

$$\begin{aligned} \nabla^2 u(P) &= -b \qquad \forall P \in \Omega \\ u(p) &= 0 \qquad \forall p \in \Gamma, \end{aligned} \tag{26}$$

where: $\Omega = \{x = (x,y) \in \Re^2 : -1 \le x \le 1, -1 \le y \le 1\}$, $b = 1$ and $\Gamma = \{x = (x,y) \in \Re^2 : x = \pm 1, y = \pm 1\}$, using a mixed finite element method and the Assembled Local Green procedure. Considering that $u_o(x,y) \in H^{3-\varepsilon}(\Omega)$, $0 < \varepsilon << 1$, the theoretical rate of convergence for finite element h-adaptivity is [5]: $\beta \ge \min(p, 2-\varepsilon)$. Due to the symmetry of the problem only 1/4 of the domain is discretized with meshes defined by the polynomial p and an index M. A mesh M = 2 and p = 1 means a 2 by 2 bilinear regular mesh.

In stating the results, the following quantities are used:

$$E_f = \frac{|u - u_o|^2_{H^1(\Omega)}}{|u_o|^2_{H^1(\Omega)}} \qquad E_u = \frac{|u - u_o|^2_{L^2(\Omega)}}{|u_o|^2_{L^2(\Omega)}}. \qquad C_f = \frac{\oint_\Gamma (u_{,n} - u_{o,n})^2 \, d\Gamma}{\oint_\Gamma u^2_{o,n} \, d\Gamma}, \tag{27}$$

where the notation $(\cdot)_{,n}$ indicates the outer normal derivative on the boundary.

From the analytical solution we obtain the following norms: $|u_o|^2_{H^1(\Omega)} = 0.5623080$, $|u_o|^2_{L^2(\Omega)} = 0.01089606$ and the contour integral of $u^2_{o,n}$ is 2.249232. Also, the value of $u_o(0,0)$ and $u_{o,x}(1,0)$ are 0.2946854 and –0.6753144 respectively. All the norms of errors shown in this problem were computed using a Gauss quadrature rule with 4 points in each direction in each element. The error in the computation of the norms will decrease with the increase of the number of elements. The maximum error, obtained for 1 bicubic element, is inferior to 2%.

Figures 3 and 4 show the behavior of the approximate solution along certain lines for coarse meshes and low order elements. Figures 5 and 6 show the experimental overall error estimates.

In general we can see close agreement for the fluxes, even for very coarse meshes, as can be expected for mixed type formulations. For bilinear elements, Figure 4, one can see clearly a tendency of exceptional precision around the middle of each boundary element. This effect was reported in early works using the Global formulation (see Silva [1988] and Barbieri [1992]). Because the results for the potential in the whole domain and fluxes on the global boundaries obtained by our Local Formulation must coincide absolutely with those results from the Global form, those effects observed here are expected. The results shown in Figure 4 show a tendency of increased error as we approximate the last element next to the corner, although we see that most of the error is concentrated in this last element. This is expected by virtue of the increase of the gradient of $u_{o,x}$ as we approach the vertices.

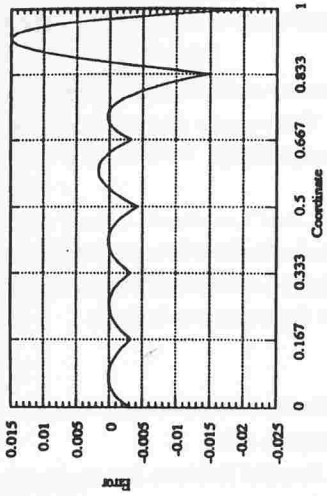

Figure 4. Absolute error ($u_{,x}$ - $u_{o,x}$) in the fluxes normal to the boundary x = 1.0. p = 1, M = 6.

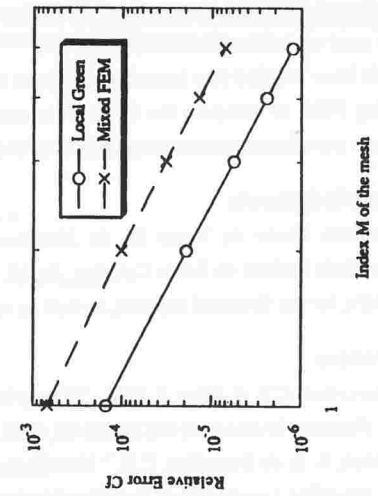

Figure 6. Error Cf of the flux normal to the boundaries. Biquadratic macrocells.

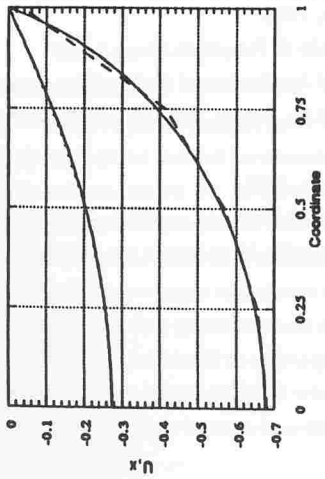

Figure 3. Normal flux along the borders x = 0.0 (—) and x = 0.5 (...). Continuous lines represent analytical solutions. M = 4, p = 1.

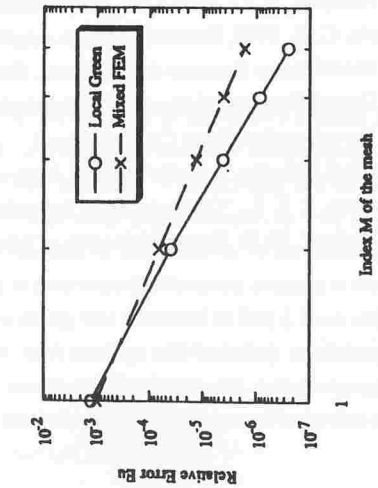

Figure 5. Error Eu of the temperature in the domain. Biquadratic macrocells.

7 - Conclusions

The MLGFM local scheme has been briefly outlined. This procedure led to: reduced computational effort; mantained the primitive variable accuracy level; and the secondary variables, referred here as fluxes, with the same level of precision as the ones computed on the boundaries by the global procedure. Among the first problems solved, typical one is presented which illustrates the convergence properties of MLGFM as compared to a mixed FE formulation which is the one in the FEM family which most resembles the MLGFM.

We have verified that boundary elements increase the accuracy and the rate of convergence, even by using FEM to compute the Green's function projections. These results suggests its use for other nonlinear continuum mechanics problems such as fluid flow and plasticity.

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GREEN'S ELEMENT METHOD- PART II: *ITERATIVE LOCAL FORM*

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ABSTRACT

An iterative procedure is developed for the computations of the secondary variables in the macrocells. Using a Somigliana's Identity and the Green's function projections computed on a microcell, it is defined a boundary integral equation involving both principal and secondary variables on its boundary. This equation, combined with the definition of the apparent fluxes and with a continuity condition across internal interfaces lead to a classical Fredholm integral of second kind for the apparent fluxes. A discretized form of this equation is used in an iterative scheme performed on sweeps over all microcells which define each macrocell. Tests are performed on one standard nonlinear heat transfer problem.

1- INTRODUCTION

In accompanying paper, Part I, we introduced the first variation of the Green's function Method. The general global method introduced by Silva, 1988, was applied to each cell independently, the cell arrays were assembled, the compatibility conditions at cell interfaces were imposed and the complete system was solved. Because the cells can be made of high order single elements as well as multiple elements and the solution can be obtained only for the tractions or fluxes normal to the boundary of the cell, it becomes necessary to use some post-processing to determine the fluxes (or stresses) in the interior of each cell. The first procedure used for this post-processing was presented in Part I. Each cell, called macrocell, was considered as an independent problem, with loadings and boundary conditions known from the first step of processing. It was divided into a new mesh of bilinear cells, called microcells, and the solution at the new internal boundaries was found using the same procedure used in the first step.