7 - Conclusions

The MLGFM local scheme has been briefly outlined. This procedure led to: reduced computational effort; mantained the primitive variable accuracy level; and the secondary variables, referred here as fluxes, with the same level of precision as the ones computed on the boundaries by the global procedure. Among the first problems solved, typical one is presented which illustrates the convergence properties of MLGFM as compared to a mixed FE formulation which is the one in the FEM family which most resembles the MLGFM.

We have verified that boundary elements increase the accuracy and the rate of convergence, even by using FEM to compute the Green's function projections. These results suggests its use for other nonlinear continuum mechanics problems such as fluid flow and plasticity.

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GREEN'S ELEMENT METHOD- PART II: *ITERATIVE LOCAL FORM*

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ABSTRACT

An iterative procedure is developed for the computations of the secondary variables in the macrocells. Using a Somigliana's Identity and the Green's function projections computed on a microcell, it is defined a boundary integral equation involving both principal and secondary variables on its boundary. This equation, combined with the definition of the apparent fluxes and with a continuity condition across internal interfaces lead to a classical Fredholm integral of second kind for the apparent fluxes. A discretized form of this equation is used in an iterative scheme performed on sweeps over all microcells which define each macrocell. Tests are performed on one standard nonlinear heat transfer problem.

1- INTRODUCTION

In accompanying paper, Part I, we introduced the first variation of the Green's function Method. The general global method introduced by Silva, 1988, was applied to each cell independently, the cell arrays were assembled, the compatibility conditions at cell interfaces were imposed and the complete system was solved. Because the cells can be made of high order single elements as well as multiple elements and the solution can be obtained only for the tractions or fluxes normal to the boundary of the cell, it becomes necessary to use some post-processing to determine the fluxes (or stresses) in the interior of each cell. The first procedure used for this post-processing was presented in Part I. Each cell, called macrocell, was considered as an independent problem, with loadings and boundary conditions known from the first step of processing. It was divided into a new mesh of bilinear cells, called microcells, and the solution at the new internal boundaries was found using the same procedure used in the first step.

In this paper, we present a second scheme which can be used either for post-processing the stresses in each individual high-order cell. Instead of assembling the arrays of all microcells of the macrocell being post-processed as before, local problems are defined for each microcell and solved iteratively in sweeps over the macrocell, giving simultaneously approximations for both the primary variables and the fluxes. Contrary to the assembled method, here the convergence is dependent on the choice of the parameter c in the definition of the apparent fluxes.

The iterative procedure for the post-processing present some advantages in accuracy when compared to both Mixed Finite Element Method and the Assembled Form described in Part I, as seen in the numerical tests performed. The formulation is developed and tested here for a nonlinear Poisson problem, although all the steps indicated are intended to be applicable to other similar boundary value problems, e.g., linear elasticity.

2 - DEFINITION OF THE NONLINEAR PROBLEM IN INTEGRAL FORM.

Let us consider the extension to the linear boundary value problem stated in Eqn. (1) of Part I:

$$\begin{aligned} -\nabla\cdot Z(u)\nabla u &= b(P), && P\in\Omega \\ u(p) &= \bar{u}(p), && p\in\Gamma_u \\ -n\cdot Z\nabla u &= f(p) = \overline{f(p)}, && p\in\Gamma_f. \end{aligned} \tag{1}$$

The matrix Z is the anisotropic nonlinear thermal conductivity of the material, decomposed here as $Z(u)_{ij} = Z_{o,ij} + \beta(u)_{ij}$, $i,j = 1..2$. All other necessary definitions in Eqn (1) are the same as those shown in Part I. Eqn. (1c) also defines the normal heat flux leaving the body. The weighted residual expression derived from (1) is (see Mendonça, 1995):

$$\begin{aligned} -\int_\Omega u\,\nabla\cdot(Z_o'\nabla w)d\Omega + \oint_\Gamma u\,(Z_o'\nabla w)\cdot n\,d\Gamma - \oint_\Gamma w(Z\nabla u)\cdot n\,d\Gamma \\ = \int_\Omega w\,b\,d\Omega + \int_\Omega w\,\nabla\cdot(\beta\nabla u)d\Omega - \oint_\Gamma w(\beta\nabla u)\cdot n\,d\Gamma. \end{aligned} \tag{2}$$

We consider the system domain Ω under consideration to be divided into cells Ω_l with boundaries Γ_l. Each of the Equations (1) to (2) can be rewritten for an individual cell, although the conditions at its boundary are *a priori* unknown. We can choose the weighting function to satisfy the Second Auxiliary Problem defined in Eqn. (5) of Part I. Therefore, a modified Somigliana's Identity similar to (6) in that paper can be found for the generic cell l:

$$\begin{aligned} \gamma u^l(q) = \sum_{\overline{m}=1}^{M}\int_{\Gamma_{\overline{m}l}} G^l(p,q)'\,F^{\overline{m}l}\,d\Gamma_p + \int_{\Omega_l} G^l(P,q)'\,[b(P)+\nabla\cdot(\beta\nabla u)]d\Omega_l \\ - \sum_{\overline{m}=1}^{M}\int_{\Gamma_{\overline{m}l}} G^l(p,q)'\,[(\beta\nabla u)\cdot n]^{\overline{m}l}\,d\Gamma_p, \quad p,q\in\Gamma_l,\ P\in\Omega_l. \end{aligned} \tag{3}$$

The boundary integrals are split for showing the contributions of each interface with the neighboring cells. $\Gamma_{\overline{m}l}$ is the interface between the cell l and a neighbor cell $\overline{m}$, and $\Gamma_l = \cup\Gamma_{\overline{m}l}$ and $\cap\,\Gamma_{\overline{m}l} = \varnothing$ for $\overline{m} = 1..M$. The *apparent flux* shown in the second term is nonlinear and is defined as

$$F^{\overline{m}l}(p) = -n^l\cdot(Z^l\nabla u^l) + c^l\,u^l(p), \tag{4}$$

where the first term is the apparent heat flux through the interface leaving the cell l to the cell m; the second term is the real nonlinear physical flow which shows on the third term in Eqn. (2).

The continuity conditions of temperature and heat flux through a generic interface l-m is

$$\begin{aligned} &f^{lm}(p) = -f^{ml}(p) \;\rightarrow\; -n^l\cdot(Z^l\nabla u^l) = n^m\cdot(Z^m\nabla u^m), \\ &u^m(p) = u^l(p) + R^{lm}\,n^l\cdot(Z^l\nabla u^l) \end{aligned} \tag{5}$$

where n^l and n^m are the unit vectors normal to the interface l-m, pointing outward the cells l and m respectively. The *Continuity Condition for the Apparent Fluxes* at the interface is:

$$F^{lm}(p) = c_1^{lm}\,u^l(p) - c_2^{lm}\,F^{ml}(p), \tag{6}$$

where $c_1^{lm} = (c^m + c^l)$, $c_2^{lm} = (c^m R^{lm} + 1)$ and R^{ml} is the contact resistance at the interface. Now we use Eqn. (3) to eliminate $u^l(p)$ from (6), obtaining an integral equation involving one sole unknown, the apparent fluxes:

$$\begin{aligned} F^{lm}(q) = \sum_{\overline{m}=1}^{M}\int_{\Gamma_{\overline{m}l}} [c_1^{lm}G^l(p,q)' - c_2^{lm}\,\delta(p,q)]\,F^{\overline{m}l}\,d\Gamma_p - c_1^{lm}\sum_{\overline{m}=1}^{M}\int_{\Gamma_{\overline{m}l}} G^l(p,q)'\,[(\beta\nabla u)\cdot n]^{\overline{m}l}\,d\Gamma_p \\ + c_1^{lm}\int_{\Omega_l} G^l(P,q)'\,[b(P)+\nabla\cdot(\beta\nabla u)]d\Omega_P, \qquad p,\,q\in\Gamma_l,\ P\in\Omega_l. \end{aligned} \tag{7}$$

To enforce uniformity in the formulation, the term F^{ml} appearing in Eqn. (6) was introduced in the first boundary integral in (7) by means of the delta function. It makes a balance between the *apparent flux* coming to the cell l from the cell m, with the net apparent flux through all faces and the source applied inside the cell. We represent the fields in the cell l by:

$$\begin{aligned} &F^{lm}(q) = [\phi^{lm}(q)]\,F^{c,lm}, && F^l(q) = [\phi(q)]\,F^l, \\ &[(\beta(u)\nabla u)\cdot n](q) = [\phi(q)]\,f_N^l, && u(P) = [\psi(P)]\,u^d, \\ &[b(P)+\nabla\cdot(\beta(u)\nabla u)] = [\psi(P)][b_o + b_N], \end{aligned} \tag{8}$$

with $q\in\Gamma_l$ and $P\in\Omega_l$. F^l is the vector with the nodal values (or generalized coefficients) of the apparent flux on the boundary of the cell l.. b_o and b_N are the vectors with the linear and nonlinear nodal values of the sources indicated in the left hand side of Eqn. (8e). The arrays $[\phi(q)]$ and $[\psi(P)]$ are properly defined arrays of basis functions, interrelated as described in (Mendonça, 1995). In particular, we have here that $[\psi(P)]\subset H^1(\Omega)$. The index lm indicate the interface l-m, and c and d indicates nodal values on boundary and domain respectively.

Applying (8) in (7), substituting the summation operator by the standard matrix-vector product, and using $[\phi(q)]$ as weighting functions we have:

$$\left[\int_{\Gamma_{lm}} [\phi^{lm}(q)]'\,[\phi^{lm}(q)]d\Gamma_q\right]F^{c,lm} = -c_1^{lm}\left\{\oint_{\Gamma_l}\left\{\int_{\Gamma_{lm}} [\phi^{lm}(q)]'\,G^l(p,q)'\,d\Gamma_q\right\}[\phi(p)]d\Gamma_q\right\}f_N^l$$

$$+ c_1^{lm}\left[\int_{\Omega_l}\left[\int_{\Gamma_{lm}}[\phi^{lm}(q)]^t G^l(P,q)^t\, d\Gamma_q\right][\psi(P)]d\Omega_P\right][b_o + b_N] \qquad (9)$$
$$+ \left\{\int_{\Gamma_l}\left\{\int_{\Gamma_{lm}}\left[c_1^{lm}[\phi^{lm}(q)]^t G^l(p,q)^t - c_2^{lm}[\phi^{lm}(q)]^t\,\delta(p,q)\right]d\Gamma_q\right\}[\phi(p)]\,d\Gamma_p\right\}F^l.$$

The terms inside each bracket can be identified with matrices defined previously. The iterative scheme described in the next section will be based on the following two algebraic equations defined for each cell l:

$$DF^c = [c_1][F][b_o + b_N] + \{[c_1][E] - [c_2][D]\}F^l - [c_1][E]f_N^l,$$
$$Du^c = EF^c + F[b_o + b_N] \qquad (10)$$

where the first expression is the matrix form corresponding to (9) and the second is Eqn. (7b) shown in Part I. The arrays $[c_1]$ and $[c_2]$ are conveniently defined diagonal matrices formed by the constants c^{lm} in each interface of the cell. Matrices D, E and F are defined in Eqs. (8) of Part I, and E and F can be eliminated in terms of D using the relations (13) in that paper. It must be noted with regard to the matrix form (10a) is that the actual operations must be made face by face of the cell, as shown in (9). The results obtained in one face are used in the computations of the next faces.

3 - SOLUTION OF THE LOCAL EQUATIONS

The general outline of the solution scheme is given below, where, to improve clarity we first consider the linear problem only, i.e., $\beta(u) = 0$, $b_N = f_N^l = 0$.

a) Choose initial distributions for the potential u^l and flux in the right hand side of Eqn. (8), for all cells; this defines the nodal vectors F^{ml} which compose F^l in Eqn. (10a);

b) Perform a sweep computing the fluxes, i.e., for each cell l do:

b1) Choose the arbitrary constants c^{lm} [later we shall discuss this procedure];
b2) Compute the arrays involved in Eqn. (10a), as well as b_o;
b3) Compute $F^{c,ml}$ for all faces of the cell;
b4) Use the continuity equation (6) to compute F^{ml};

c) Test the errors by computing the difference between the fluxes obtained in the last two iterations;

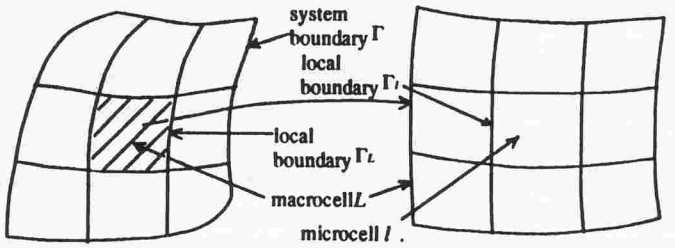
d) Perform a sweep computing the temperatures using Eqn. (10b); Test errors in the temperatures;

Clearly, the local arrays can be computed just once and left stored for the subsequent iterations. The values $F^{c,lm}$ obtained when computing the cell l are used in the same sweep when computing the next cell in the same sweep. Because the computations are local, the values obtained are discontinuous, making necessary nodal averaging. In case of nonlinearity, the local iterations above are performed for each pair b_N, f_N^l. After convergence, these vectors are updated and new local iterations are performed.

The initial distribution in step a) for the Local Iteration can be defined as the converged values of the last nonlinear iteration.

4.4 ITERATIONS IN THE MACROCELLS

In this section we give an overview of the steps necessary in the complete analysis and the use of the formulation described above as a post-processor.



a) Discretization of the system domain Ω into macrocells L.

b) Discretization of the macrocell L into microcells l.

Figure 1 - Macro and microcells.

In the first step of the analysis, the system domain and boundary are discretized in macrocells as depicted in Figure 4.1a. The primary and secondary unknown variables, temperature and normal fluxes respectively in potential problems, are computed at the local boundaries Γ_L of the macrocells, using the "Assembled Local Form" presented in Part I. The complete solution will require a second step of processing, unless the macrocells are composed by single bilinear quadrilaterals or linear triangular elements. Therefore, each macrocell is taken individually as an isolated problem and discretized in subcells as shown in Figure 1b. This problem is then solved using the local iterative procedure presented in the first Sections of this article.

In this secondary analysis, the boundary conditions for each macrocell are known from the solution of the first step. But we notice that, since the first step of processing produced both temperatures and normal fluxes on Γ_L, in the second step we are in the unusual situation of having available both magnitudes to use as boundary conditions. Due to the iterative nature of the procedure, this is readily incorporated into the scheme.

In the case where part of the local boundary Γ_L belongs to the global boundary Γ_u, the temperature values given there as boundary conditions for the second step of the analysis will be exact, because they are the exact boundary conditions of the problem; consequently only the fluxes there are approximate, because they were computed in the first step of the analysis. The second case, where part of Γ_L belongs to Γ_f, the discussion is analogous, except for interchanging of the words "temperature" and "normal fluxes".

The third case happens when the macrocell is completely interior to the system domain, i.e., $\Gamma_L \cap \Gamma = \emptyset$. Here temperature and normal fluxes on Γ_L are both approximated. There is no need to choose arbitrary initial values for the apparent fluxes on the boundary before the first iteration. Instead, we use Eqn. (4) to compute $F^u(p) = c^l u^l(p) - \bar{f}(p)$, $p \in \Gamma_L$, and then, considering absence of contact resistance, we have that (see Mendonça, 1995) $F^u(p) = F^u(p) + 2\bar{f}(p)$, $p \in \Gamma_L$, where the right hand side of both expressions are now entirely known from the results of the first step of processing.

5 - NUMERICAL RESULTS

The test codes developed and the models run have the following characteristics:

- Macrocells are composed by a single Lagrangian element of polynomial order $p \leq 3$;
-Each macrocell can be divided in meshes of up to 5x5 bilinear microcells;
-The nonlinear iterative method used was the simple Successive Iteration Method; it is the method widely used for Boundary Element Method type of programs for mildly nonlinear problems. Besides, it does not complicate unnecessarily a test program;
-All results were compared to those obtained by the Mixed Element formulation described in (Mendonça, 1995) and to the analytical solutions. The same nonlinear iterative technique and convergence test were used for both Green and Mixed Finite Element test programs. In all models the value chosen for c was the same for all cells.

In the Figures shown next, the following notation is used:

Ef, Eu	- Relative error of the temperature and flux in the domain, defined in Eqs. (27) in Part I;
ALG	- *Assembled Local Green* ; Indicates results obtained in the solution of the Problem 1 in Part 1;
ILG	- *Iterative Local Green* Method. Indicates results obtained by the procedures described in the present paper;
mcl, Mcl	- microcell and Macrocell respectively;
NITL, NITN	- Number of Local Iterations and Number of Nonlinear Iterations respectively;
M, p	- Mesh parameter. Indicates a regular MxM mesh and p is degree of the polynomial used;
Rr	- Relative Residue in the nonlinear iterations, given by $Rr = \|R_{NITN} - R_{NITN-1}\|^2 / \|R_{NITN}\|^2$.

The vector R is the residue in the algebraic nonlinear equations. These iterations are performed until $Rr \leq eps$. After each iteration is performed, the solution is used to compute the updated source b_N as implicitly defined in Eqn (8) and the residue Rr.

5.1 - PROBLEM 1

In this test we will evaluate the behavior of the scheme to solve the standard linear problem defined in (1), with $Z = I$, $b = -1$, $\Gamma_u = \Gamma$ and $\bar{u} = 0$ on a square of sides 2x2.

Figure 2 shows the errors Ef of the flux for a mesh of 4x4 bilinear macrocells, with each one modeled by a mesh of 2x2 microcells, as a function of NITL for two different values of c. Here we see also a general tendency: the error first drops quickly in the first or second iterations and then stabilizes in a nearly flat path.

Figures 3 to 5 show the relative error Ef for the flux in the domain at the Local Iterations 2 and 4 as function of c, for meshes of biquadratic and bicubic macrocells. In these Figures we can see some general tendencies followed by all studied:

- There is a value of c for which the error for the ILG is minimum;
- The curve Ef x c is relatively shallow, revealing low sensitivity of the results with c;
- The error Ef for ILG is considerably smaller than the errors obtained either by ALG or Mixed FEM;

Observing Figures 2 and 3, we see that the value of c for minimum errors, c_M changes considerably with the mesh index M, therefore we make a first effort in trying to produce an estimator c_{ref} to localize at least roughly the region of lower errors. Observing other results (Mendonça, 1995) for M = 1 to 4 respectively, we see that c_M is different for each Local Iteration NITL. Therefore, to simplify the task, we will elect the second Local Iteration as our goal. For the biquadratic macrocells we have chosen the following estimator

$$c_{ref} = \frac{\sum_i |K_{o,1i}|}{\sum_i |K_{3,1i}|}. \qquad (12)$$

where the matrices K_o and K_3 are defined in (Mendonça, 1995). Figures 3 to 5 show good agreement for different macromeshes, and for 2x2 and 3x3 microcells per macrocell. Other results also show that c_{ref} gives good indications not only for the small error zone of the fluxes, but also for the potential. Due to the small sensitivity and fast rates of convergences, c_{ref} seems to indicate correctly the beginning of the low error regions for Iterations 3 and 4 too. This feature seems very convenient, since we do not intend to use many more than two iterations

5.2 - PROBLEM 2

In this problem we want to approximate the solution to the nonlinear extension of Problem 1. The nonlinearity is introduced by using a thermal conductivity in the form $k(u) = a_o + a_1 u$. The source term is defined by $-a_o b(x,y) = a_1 (u_{,x}^2 + u_{,y}^2) + (a_o + a_1 u)\nabla^2 u$, where

$$a_1^2 u(x,y) = (p_x^{1/2} - a_o)(p_y^{1/2} - a_o), \qquad a_1 u_{,x}(x,y) = -g\, x\, p_x^{-1/2}(p_y^{1/2} - a_o)$$
$$p_x = a_o^2 + a_1 g (l_x^2 - x^2), \qquad p_y = a_o^2 + a_1 g (l_y^2 - y^2) \qquad (13)$$

The function $u(x,y)$ in Eqn. (13) not only defines the source term, but it is also the solution for the problem in a rectangle with sides $2l_x \times 2l_y$, with $u = 0$ at the boundaries. For the computations we have chosen $g = l_x = l_y = 1.0$. The material property constant a_o was set equal to 1.0. The material constant a_1 which gives account of the nonliearity was chosen to assume the values 0.1, 0.5 and 1.0 in the tests.

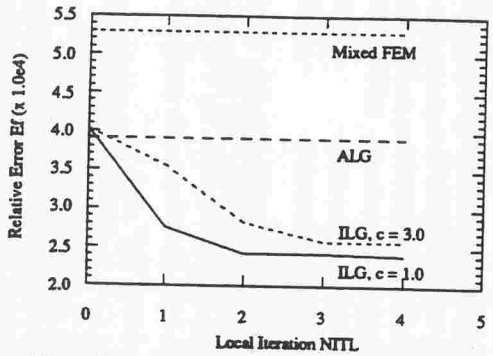


Figure 42. Error Ef of the the flux for M = 4, p = 1, 2x2 mcl/Mcl.

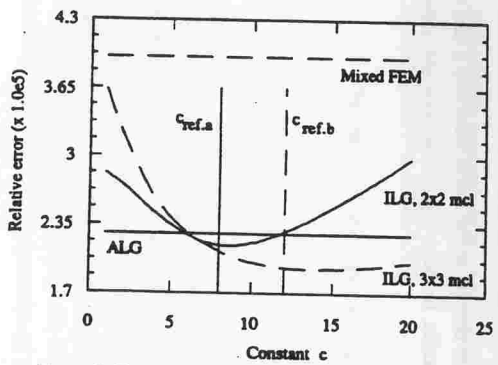


Figure 3. Error Ef of the flux for M = 3, p = 2. $c_{ref.a}$ = 8.00 and $c_{ref.b}$ = 12.00 for 2x2 and 3x3 mcl respectively. NITL 2.

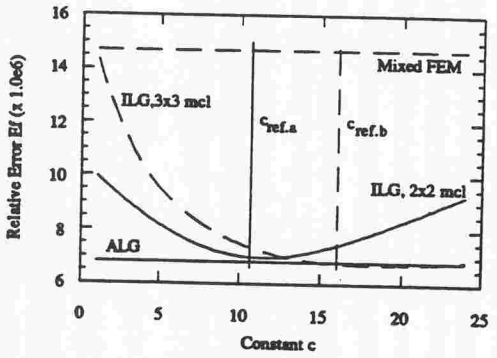


Figure 4. Error Ef of the the flux for M = 4, p = 2. $c_{ref.a}$ = 10.67 and $c_{ref.b}$ = 16.00 for 2x2 and 3x3 mcl respectively. NITL 2.

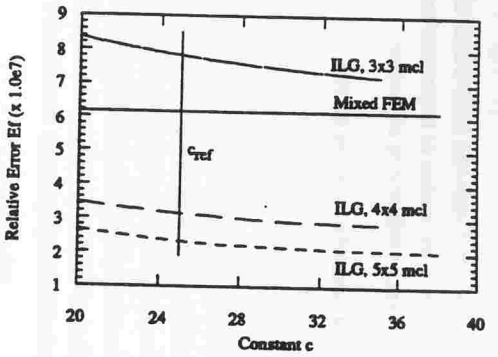


Figure 5. Error Ef of the flux for 4x4 bicubic macrocells,

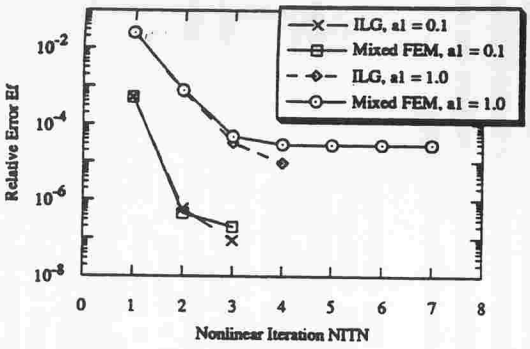


Figure 6. Error Ef of the flux for M = 4, p = 2, 3x3 mcl. Maximum NITL = 3. The nonlinear iterations stop when Rr < 1.0e-10.

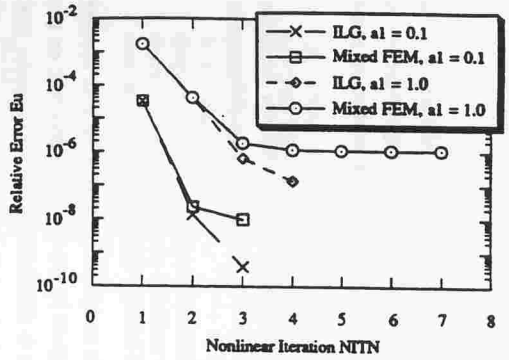


Figure 7. Error Eu of the flux for M = 4, p = 2, 3x3 mcl. Maximun NITL = 3. The nonlinear iterations stop when Rr < 1.0e-10.

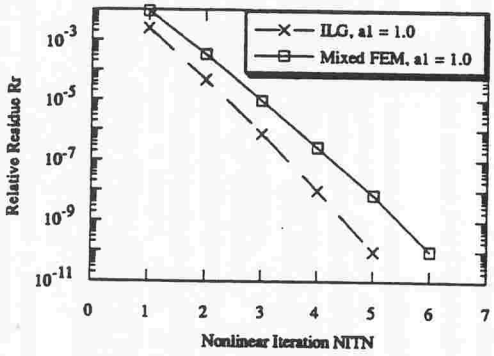


Figure 8. Relative Residue Rr for M = 2, p = 2, 3x3 mcl. Maximum NITL = 3. The nonlinear iterations stop when Rr < 1.0e-10.

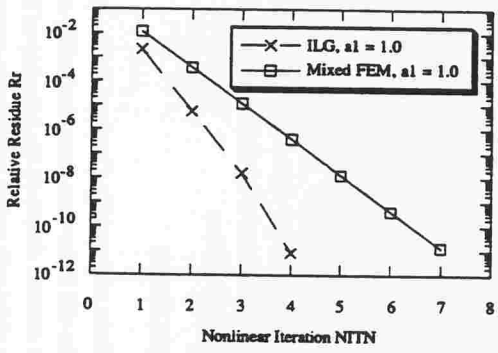


Figure 9. Relative Residue Rr for M = 4, p = 2, 3x3 mcl. Maximu NITL = 3. The nonlinear iterations stop when Rr < 1.0e-10.

Figures 6 and 7 show the relative errors in flux and potential, Ef and Eu, along the number of nonlinear iterations NITN, for different meshes and values of a_1. All results for the Iterative Local Green Method were obtained using the reference value for c, as defined in Eqn. (12). These values are: c_{ref} = 4.0, 6.0, 12.0 and 16.0 for M = 1,2,3 and 4 respectively. Also, in all cases, the maximum number of Local Iterations NITL was set to 3.

In Figures 6 and 7 we see that the number of nonlinear iterations necessary for convergence was considerably different between the ILG and the Mixed FEM. Looking carefully at the logarithmic scales we see that not only the convergence was reached earlier for ILG, but the error was smaller, in some cases by one order of magnitude.

Figures 8 and 9 show the evolution of the Relative Residue *Rr* along the Nonlinear Iterations NITN for both methods, for meshes of 2x2 and 4x4 macrocells. We notice an accentuated difference in the rate of convergence.

6 - CONCLUSION

Tests performed in Chapter 4 for the potential and fluxes computed iteratively by ILG showed accuracies still better than those obtained by ALG. The results given by the ILG are dependent on the arbitrary constant chosen on the boundary of the microcells; this is different than the ALG results The matrices of the cells are not assembled to represent the whole physical system, but instead they are solved iteratively in sweeps. This process of solution introduces the inconsistency in the which leads to the dependence of the results with the constants. An empirical formula to obtain a gross estimate for the optimal value of this constant was given and tests showed acceptable agreement. Tests also showed that the accuracy dependence of this constant was not strong, making it sufficient to use a value inside an optimal region of values instead of the optimal point itself. Also, results showed that, for limited number of microcells and for the constant not too far from the optimal region, the number of iterations can be limited to the range 2 to 4, and can be set by the analyst *a priori*, in the definition of the data.

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CONGRESSO IBERO LATINO AMERICANO SOBRE MÉTODOS COMPUTACIONAIS PARA ENGENHARIA

IBERIAN LATIN AMERICAN CONFERENCE ON COMPUTATIONAL METHODS FOR ENGINEERING

MÉTODO DE SOLUÇÃO NUMÉRICA DE ESCOAMENTOS INCOMPRESSÍVEIS EM GEOMETRIAS COMPLEXAS

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SUMÁRIO

Neste trabalho é apresentado um algoritmo para a solução numérica de problem de escoamento de fluidos incompressíveis em sistema de coordenadas curvilíneas nã ortogonais generalizadas, utilizando a técnica de volumes finitos. O problema acoplamento pressão-velocidade é solucionado usando uma combinação das equações continuidade e da quantidade de movimento linear, gerando duas equações de correção pressão. Resultados da aplicação do método são apresentados.

1. INTRODUÇÃO

Métodos numéricos para a solução de problemas envolvendo escoamento de fluid em geometrias regulares estão bem desenvolvidos atualmente, porém na maioria d aplicações em engenharia as geometrias encontradas são complexas. Estimulado pe desenvolvimento de vários métodos de geração de malhas curvilíneas que se ajustam a formatos dos contornos, novos algoritmos passaram a ser desenvolvidos usando u sistema de coordenadas não-ortogonais generalizadas, permitindo o uso de malh adaptáveis a geometria do domínio físico do problema a ser solucionado. Os seguint trabalhos em coordenadas generalizadas podem ser citados: (Maliska, 1984); (Peric,198 (Shyy et al.,1985); (Reggio e Camarero, 1986); (Hadjisophocleous et al, 1988); (Karki Patankar, 1988); (Davidson e Hedberg, 1989); (Thangam e Knight, 1990); (Yang et 1990); (Deng et al, 1991); (Kelkar e Choudhury, 1991); (Silva, 1991); (Pires, 199 diferenciando-se entre si, basicamente, pela configuração de malha utilizada (co-localiza ou deslocada), e do tipo e local dos componentes da velocidade. Essas distinções s amplamente discutidas por Shyy e Vu (1991).

Este trabalho dá prosseguimento ao apresentado por Ronzani e Nieckele (1994) propõe um algoritmo para a solução numérica de problemas de escoamento de fluid incompressíveis em coordenadas curvilíneas bidimensionais. A técnica de volumes finitos usada, com uma configuração de malha co-localizada para todas as variáveis.

Figures 6 and 7 show the relative errors in flux and potential, Ef and Eu, along the number of nonlinear iterations NITN, for different meshes and values of a_1. All results for the Iterative Local Green Method were obtained using the reference value for c, as defined in Eqn. (12). These values are: c_{ref} = 4.0, 6.0, 12.0 and 16.0 for M = 1,2,3 and 4 respectively. Also, in all cases, the maximum number of Local Iterations NITL was set to 3.

In Figures 6 and 7 we see that the number of nonlinear iterations necessary for convergence was considerably different between the ILG and the Mixed FEM. Looking carefully at the logarithmic scales we see that not only the convergence was reached earlier for ILG, but the error was smaller, in some cases by one order of magnitude.

Figures 8 and 9 show the evolution of the Relative Residue *Rr* along the Nonlinear Iterations NITN for both methods, for meshes of 2x2 and 4x4 macrocells. We notice an accentuated difference in the rate of convergence.

6 - CONCLUSION

Tests performed in Chapter 4 for the potential and fluxes computed iteratively by ILG showed accuracies still better than those obtained by ALG. The results given by the ILG are dependent on the arbitrary constant chosen on the boundary of the microcells; this is different than the ALG results The matrices of the cells are not assembled to represent the whole physical system, but instead they are solved iteratively in sweeps. This process of solution introduces the inconsistency in the which leads to the dependence of the results with the constants. An empirical formula to obtain a gross estimate for the optimal value of this constant was given and tests showed acceptable agreement. Tests also showed that the accuracy dependence of this constant was not strong, making it sufficient to use a value inside an optimal region of values instead of the optimal point itself. Also, results showed that, for limited number of microcells and for the constant not too far from the optimal region, the number of iterations can be limited to the range 2 to 4, and can be set by the analyst *a priori*, in the definition of the data.

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