AN ERROR ESTIMATOR FOR ADAPTIVE FRICTIONLESS CONTACT FINITE ELEMENT ANALYSIS

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ABSTRACT

Computationally efficient adaptive procedures for the numerical solution of variational inequalities of elliptic type, which arise e.g. in frictionless elastic contact problems, have received special attention over the last years. This is because powerful mathematical programming algorithms have become available, together with efficient numerical methods and their integration with solid modeling, visualization of engineering data and automatic mesh generation.

In any adaptive procedure, *a posteriori* error estimators play an important role in the process of assessing the accuracy of the approximate solution. Based on the information given by these estimators, it is possible to decide whether the adaptive process must be stopped or, if this is not the case, where and how mesh refinement might be performed more efficiently.

In the linear case, several approaches are available to define error estimators for different problems using the residual equation. To extend these techniques to variational inequalities, the main difficulty is that the error is not orthogonal to the set of approximate functions. This feature yields terms in the error equation that depend on the exact solution and cannot be neglected. Local *a posteriori* error estimators for variational inequalities have been proposed by Ainsworth et al. and applied to the obstacle problem. Following a different approach, Johnson also reports an adaptive finite element method for the same problem. We have used Johnson's ideas to derive an *a posteriori* error estimator for the frictionless contact problem, which differs from the obstacle problem in that an inequality constraint must hold at the boundary of the domain instead of in its interior. This error estimator is then used in adaptive finite element solution of test problems to assess the reliability and computational efficiency of this estimator.

The presentation is organized as follows: In Section 2, the primal formulation of the mathematical model and its optimality conditions are briefly reviewed. The penalization technique and finite element approximation are also included. Based on the penalized approach, an *a posteriori* error estimator is proposed in Section 3. It is also proved in this section that this estimator provides an upper bound for the discretization error. Numerical evidence that the optimal order of convergence is obtained with an adaptive procedure based on this estimator and its comparison with other *a posteriori* error estimators in the literature are provided through several numerical experiments in Section 4.