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OPTIMIZATION OF THE RESPONSE ON THE FREQUENCY DOMAIN ON COMPOSITE ROTORS

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Abstract. The purpose of this work is to investigate the response on the frequency domain on rotors in wounding shaft. The finite element method is used and the mechanical properties of the shaft are obtained using an equivalente module. The internal damping due to composite materials is introduced in the analysis by using a model proposed by Adams frequently used on laminate strutures. The disks are considered rigids and the bearings are flexible supposed be isotropics or anisotropics. The external excitation can also be synchronous or unsynchronous. An unconstrained optimization technique is used in order to minimize the amplitude of vibration by maximizing the viscous loss factor on the first critical speed. In theses cases the design variables considered are the wounding angles, the stiffness of the bearings and the position of the disk.

Keyword:. *rotors, composite materials, optimization, response, finite element*

1. Indroduction

The anisotropic properties of the composite materials can be widely explored on the prediction of the dynamic behavior of rotors in wounding shaft. This aspect including the weight and the internal damping can be used on searching for the optimal design of the rotor. Considering that for a wide range of rotors the operating rotation is higher than the first critical velocity, it is important to reduce the amplitude of vibration when passing through this velocity. In this case the laminated composite materials has an enormous advantage in comparision to the metalic materials. The matrix of the composite materials has higher capacity of damping than the isotropic materials.

The internal damping in composite materials have been recently introduced in rotodynamic analysis by Wettergren (1996), Gupta et all. (1998) and Silveira (2001) where the damping model is similar to the one proposed by Adams et all. (1973).

The internal damping in the case of rotordynamics is a hysteretic damping and can be treated as a viscous damping by using an equivalence between the energy dissipated by both mechanisms, Singh et all. (1994). Zorzi et all. (1977) and Özgüven et all. (1984) have incorporated the hysteretic and the viscous damping in rotor problems in order to investigate the instability regions on rotors in isotropic materials. Silveira (2001) has also included the hysteretic damping on analysing the instability and the response on frequency on rotors in wounding shaft.

In this work the finite element method is used to analyse the dynamic behavior of composite rotors. The shaft is obtained by winding several layers of embbebed fibers over a mandrel. The disk is supposed to be rigid and the assembly is supported by flexible bearings. An equivalent module approach is used in order to represent the orthotropic properties of the composite shaft. The effect of the damping in composite materials is made by introducting a model proposed by Adams et all. (1973). A strain stress relation is developed in order to include the internal damping on the strain energy in bending. The purpose of this work is to analyse the effect of the internal damping of the composite materials on the unbalance response of rotors by using optimization techniques. In this work the wounding angle, the stiffness of the bearings and the position of the disk are used as design variables.

2. The finite element model

The finite element model of a rotor is composed by beam elements and rigid elements to represent the shaft and the disk respectively. The rotor is supposed to be simple-supported and the wounding angle of each layer of the shaft is φ.

As shown by Lalanne et all. (1998), the kinetic energy of the disk can be expressed by:

$$T_D = \frac{1}{2} M_D \left(\dot{u}^2 + \dot{w}^2 \right) + \frac{1}{2} I_{Dx} \left(\dot{\theta}^2 + \dot{\psi}^2 \right) + I_{Dy} \Omega \dot{\psi} \theta + \frac{1}{2} I_{Dy} \Omega^2 \qquad (1)$$

where M_D is the mass of the disk, u and w are the coordinates of the center of inertia of the disk on the inertial axis, I_{Dx} and I_{Dy} are the moments on the principal directions of inertia. The rotation of the rotor is Ω and $\dot{\psi}$ and $\dot{\theta}$ are instantaneous velocities.

For an element of the shaft, the kinetic energy can be expressed by:

$$T_S = \frac{\rho S}{2}\int_0^L \left(\dot{u}^2 + \dot{w}^2\right)dy + \frac{\rho I_{xx}}{2}\int_0^L \left(\dot{\theta}^2 + \dot{\psi}^2\right)dy + \rho I_{xx} L \Omega^2 + 2\rho I_{xx}\Omega \int_0^L \dot{\psi}\theta dy \tag{2}$$

where ρ is the volumetric mass, S is the area of the cross section, I_{xx} is the inertia moment of the cross section and L is the lenght of the element.

The general expression for the strain energy of the shaft in bending is:

$$U = \frac{1}{2}\int_V \{\varepsilon\}^t [\sigma] dV \tag{3}$$

As proposed by Silveira (2001), the stress-strain relation for a composite beam, including the effect of hysteretic damping can be given as:

$$\sigma = E_{eq}\varepsilon + E_{eq}^{\psi}\dot{\varepsilon} \tag{4}$$

where E_{eq} is the equivalent Young's module and E_{eq}^{ψ} is the equivalent damped Young's module. Considering small deformations, the longitudinal strain and the longitudinal strain rate can be expressed as:

$$\begin{aligned} \varepsilon &= -x\frac{\partial^2 u^*}{\partial y^2} - z\frac{\partial^2 w^*}{\partial y^2} \\ \dot{\varepsilon} &= -x\frac{\partial^2 \dot{u}^*}{\partial y^2} - z\frac{\partial^2 \dot{w}^*}{\partial y^2} \end{aligned} \tag{5}$$

where u^* and w^* are displacements of the geometric center measured on the rotating axis of the shaft, Lalanne et all. (1998). Considering the relation between the displacements u^* and w^* and the displacements u and w measured on the inertial axis, Lalanne et all. (1998), and using Eqs. (3)-(4), we obtain the strain energy in bending:

$$\begin{aligned} U &= \frac{1}{2}E_{eq}I_{xx}\int_0^L \left[\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + \left(\frac{\partial^2 u}{\partial y^2}\right)^2\right]dy + \frac{1}{2}E_{eq}^{\psi}I_{xx}\int_0^L \left[\left(\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 \dot{u}}{\partial y^2}\right) + \left(\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 \dot{w}}{\partial y^2}\right)\right]dy \\ &+ \frac{1}{2}E_{eq}^{\psi}I_{xx}\int_0^L \left[-\Omega\left(\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 w}{\partial y^2}\right) + \Omega\left(\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 u}{\partial y^2}\right)\right]dy \end{aligned} \tag{6}$$

The second term and the third term of Eq. (6) are related to the hysteretic damping named $[H_b]$ and $[H_c]$. The equivalence between the hysteretic damping and the viscous damping is made by:

$$\begin{aligned} [K_b] &= \frac{[H_b]}{\pi[w]} \\ [K_c] &= \frac{[H_c]}{\pi[w]} \end{aligned} \tag{7}$$

where $[w]$ is the diagonal frequencies matrix.

The equation of motion of the rotor is obtained by applying Lagrange's equations on the kinetic energy and the strain energy of the elements:

$$[M]\{\ddot{u}\} + [K_b + \Omega G]\{\dot{u}\} + [K + \Omega K_c]\{u\} = \{F\} \tag{8}$$

where $[M]$, $[G]$ and $[K]$ are global mass, global Coriolis and global stiffness matrices. $[K_b]$ and $[K_c]$ are global dissipation matrix and global circulation matrix. Vectors $\{\ddot{u}\}$, $\{\dot{u}\}$, and $\{u\}$ are nodal acceleration, nodal velocity and nodal displacement respectivelly and $\{F\}$ is the generalised force vector due to the unbalanced mass. The elementary matrices are obtained according to the Euler-Bernoulli equation for beams and are presented in Zorzi et all. (1977).

3. The equivalent module and the internal damping model

Considering that the shaft is thin walled and slender, and the laminate is symmetric and balanced, the equivalent Young's module is presented as, Singh et all. (1994):

$$E_{eq} = \frac{\left[4(U_1 - U_5)(U_5 + U_3\gamma) - \beta^2 U_2^2\right]}{U_1 - \beta U_2 + \gamma U_3} \tag{9}$$

where $U_{1\text{-}5}$ are the laminate invariants, Tsai et all. (1980) and:

$$\gamma = \sum_{k=1}^{N} \frac{h_k}{h}\cos(4\varphi_k) \qquad \beta = \sum_{k=1}^{N} \frac{h_k}{h}\cos(2\varphi_k) \tag{10}$$

where h_k is the thickness of layer k, h is the laminate thickness and φ_k the wounding angle.

On the prediction of damping on multi-layer shell structure the model proposed by Adams is used and the specific damping capacity is defined as:

$$\psi = \frac{\Delta U}{U} \tag{11}$$

From Eq. (3) and Eq. (11) and assuming plane stress state, the dissipative energy for a single layer of unidirectionally fiber in the orthotropic axis is:

$$\Delta U = \frac{1}{2}\int_V \{\varepsilon\}^t[\psi]\{\sigma\}dV \tag{12}$$

where $[\psi]$ is the specific damping capacity matrix in the form:

$$[\psi] = \begin{bmatrix} \psi_{11} & 0 & 0 \\ 0 & \psi_{22} & 0 \\ 0 & 0 & \psi_{12} \end{bmatrix} \tag{13}$$

where ψ_{11}, ψ_{22}, ψ_{12} are the specific damping capacity of a layer on longitudinal, transverse and shear direction. From Eq. (12) and considering the constitutive relation, we obtain:

$$\Delta U = \frac{1}{2}\int_V \{\varepsilon\}^t[\psi][Q]\{\varepsilon\}dV \tag{14}$$

Using the same procedure to derive E_{eq}, the equivalent damped Young's module E_{eq}^{ψ} as a function of the specific damping is derived from Eq. (14).

4. The unbalance response

The unbalance is defined by a mass m_u situated at a distance d from the geometric center of the shaft. The general equation of the rotor with this excitation become, Lalanne et all. (1998):

$$[m]\{\ddot{q}\} + [c]\{\dot{q}\} + [k]\{q\} = \{f_1\}\sin\Omega t + \{f_2\}cos\Omega t \tag{15}$$

where $\{f_1\}$ and $\{f_2\}$ are related to m_u, d and Ω^2, and [m], [c] and [k] are the modal matrices obtained from Eq. (8) by using the pseudo-modal method, Lalanne et all. (1998). Solutions for this problem are sought as:

$$\{q\} = \{p_1\}\sin\Omega t + \{p_2\}cos\Omega t \tag{16}$$

and the identification of the terms in $sin\Omega\, t$ and $cos\Omega\, t$ gives the equation:

$$\begin{bmatrix} k - m\Omega^2 & -\Omega c \\ \Omega c & k - m\Omega^2 \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} \tag{17}$$

The system of the Eq. (17) is solved and the unbalance response is determined for any values of Ω. The response to an unsynchronous excitation is solved on this same way where the frequency of the excitation is $s\Omega$ for $s\neq1$.

5. Applications

A non-linear unconstrained optimization technique was used for minimize to peak of amplitude on the first critical speed by the maximization of the loss factor in the same rotation. It was used the Quasi-Newton method, where the approach of the Hessian is made by the BFGS, Arora (1989), and the gradient of the objective function was determined by the forward finite difference.

As observed by Özgüven et all. (1984) and Silveira (2001) the internal damping has no effect on the response to a synchronous excitation for rotors supported by isotropic bearings. So, only rotors with isotropic bearings unsynchronously excitated and rotors with anisotropic bearings synchronously excitated were analysed.

Firstly, in order to emphasize the influence of the internal damping due to composite materials in rotordynamics analysis, the response to an unbalance mass is determined for a rotor supported by anisotropic bearings. In this case, the rotor is composed by a wounding shaft, two disks equidistants from the ends and the assembly is supported by flexible bearings, Fig. (1).

The wounding shaft has length of 1.2m, inner radius of 0.04m, outer radius of 0.048m, with eight layer of 0.001m thickness in a balanced and symmetric configuration such as $[\pm\varphi]_s$. The disks have inner radius of 0.048m, outer radius of 0.15m and thickness of 0.05m. The stiffness of bearing are given as K_{xx} = 1.10^7 N/m, K_{zz} = 1.10^8 N/m. A mass of 10^{-4}Kg was placed at 0.15m from the center of the first disk and the response is taken on the node corresponding to the first disk. The material data are given in Tab.(1).

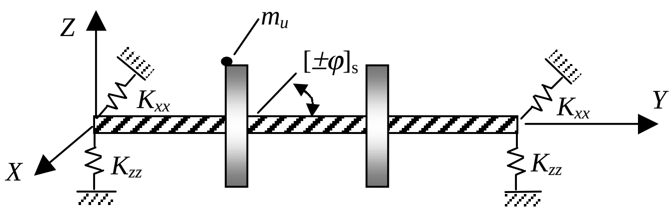


Figure 1. Rotor in wounding shaft with two disks.

Table 1. Material data of the shaft and the disks.

	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ρ (kg/m^3)	ψ_{11} (%)	ψ_{22} (%)	ψ_{12} (%)	ν_{12}
Shaft (carbon/epoxy)	172.7	7.20	3.76	1446.2	0.45	4.22	7.05	0.3
Shaft (glass/epoxy)	37.78	10.90	4.91	1813.9	0.87	5.05	6.91	0.3
Disk (steel)	-	-	-	7800	-	-	-	-

Figures (2), (3) and (4) show the influence of the wounding angle on the position of the critical speed, as well as the influence of the internal damping on the peak of amplitude for rotors in carbon/epoxy, supported by anisotropic bearings. It can be observed that, as major is the wounding angle, minor are the natural frequencies of the rotor, because of the equivalent stiffness of shaft decrease with the increase of wounding angle.

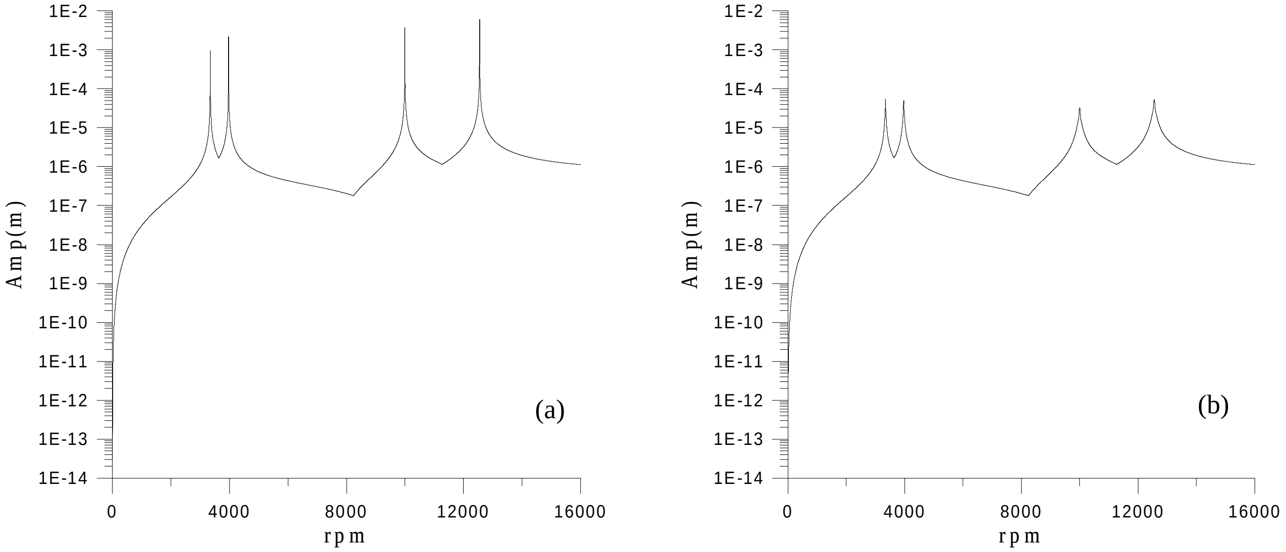


Figure 2. Response to an unbalance mass for $\varphi = 15º$, (a) without internal damping; (b) including internal damping.

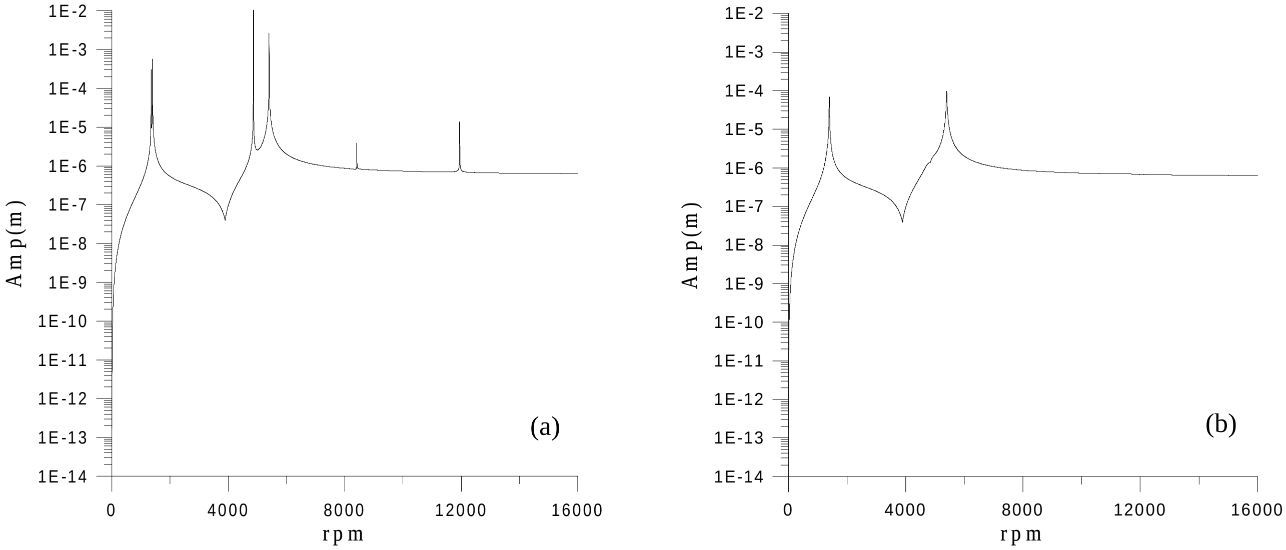


Figure 3. Response to an unbalance mass for $\varphi = 45º$, (a) without internal damping; (b) including internal damping.

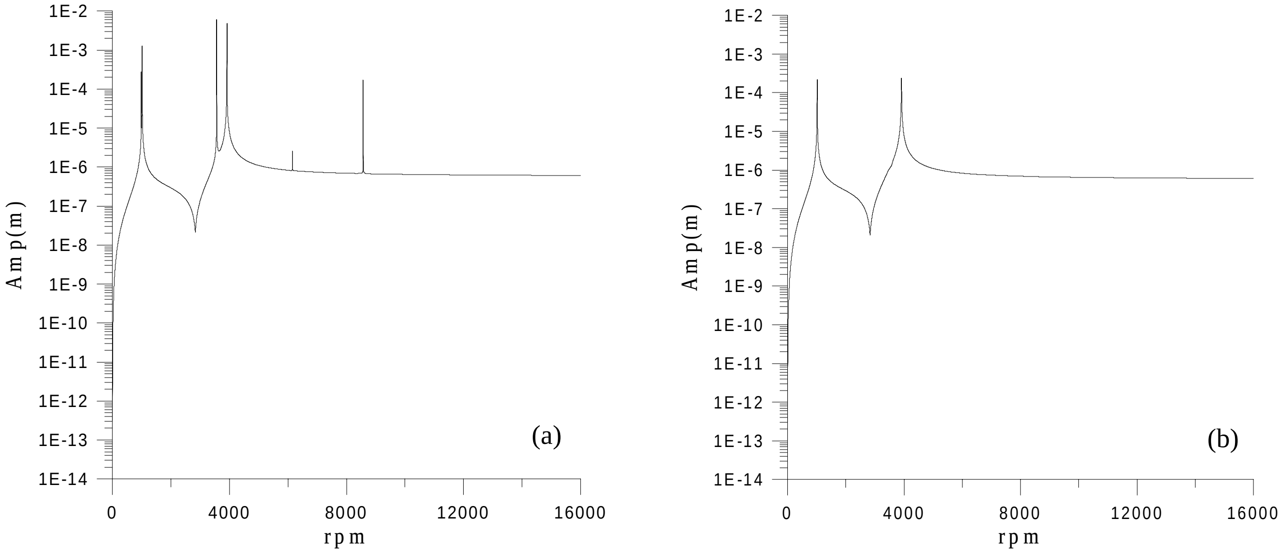


Figure 4. Response to an unbalance mass for $\varphi = 75º$, (a) without internal damping; (b) including internal damping.

5.1. Optimization on rotors with unsynchronous excitation

In this case, the rotor is unsynchronously excitated, $\omega = 2\Omega$, and is supported by isotropic bearings. The problem of optimization was formulated as:

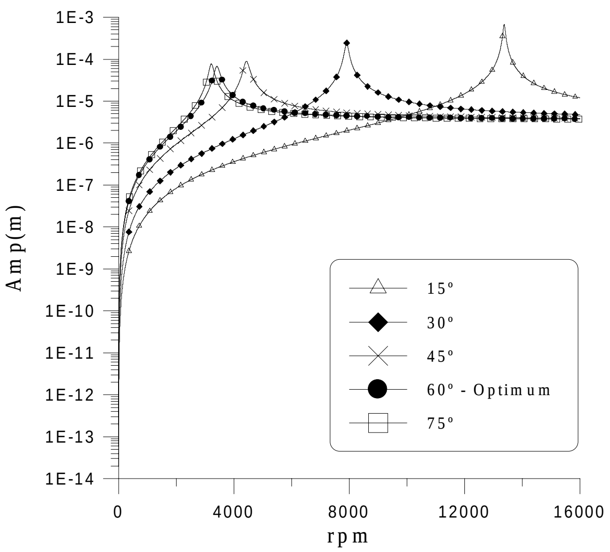
$$\text{Max } \eta_i$$
$$15^\circ \leq \varphi_1 \leq 75^\circ$$
$$15^\circ \leq \varphi_2 \leq 75^\circ$$

where η is the loss factor evaluated only on the first critical speed.

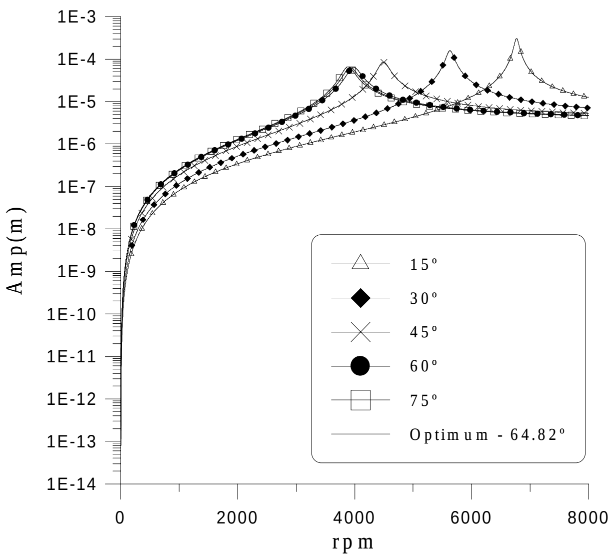
The rotor has now only one disk located at 0.33m from the end. The wounding shaft has length of 1m, inner radius of 0.031m, outer radius of 0.039m, with eight layer of 0.001m thickness in a balanced and symmetric configuration as $[\pm\varphi_1, \pm\varphi_2]_s$. The disk has inner radius of 0.039m, outer radius of 0.15m and thickness of 0.03m. The stiffness of bearing are given as $K_{xx} = K_{zz} = 8.10^7$N/m. The response is taken on the node corresponding to the disk. The solution for rotors in carbon/epoxy and glass/epoxy are shown in the Fig. (5) and (6). The results are shown on Tab. (2).

Table 2. Optimal solution for rotors with unsynchronous excitation

Optimal solution		
	φ_1	φ_2
Carbon/epoxy	60,19º	60,19º
Glass/epoxy	64,82º	64,82º

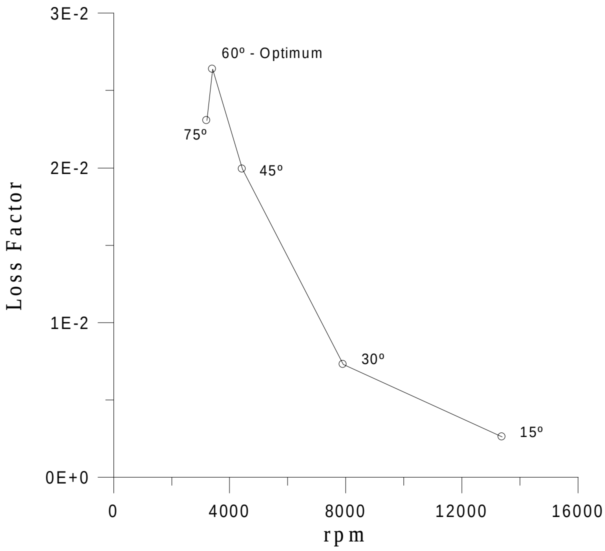


(a)



(b)

Figure 5. Response to an unsynchronous excitation for rotors in: (a) carbon/epoxy; (b) glass/epoxy.



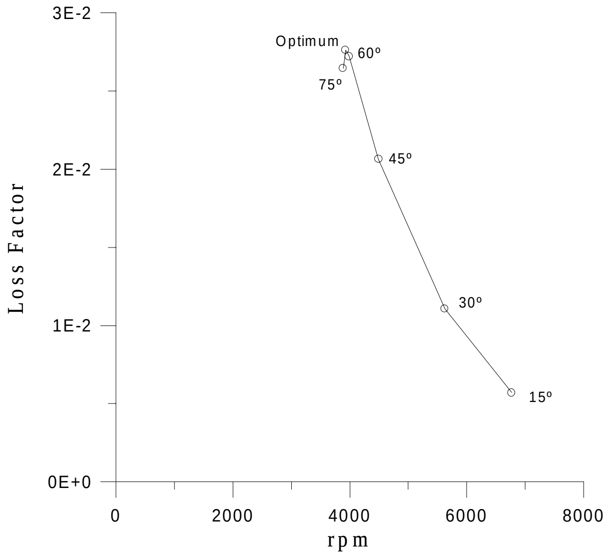


Figure 6. Loss factor on the first crítical speed: (a) carbon/epoxy; (b) glass/epoxy

5.2. Optimization on rotors with synchronous excitation and anisotropic bearings

The rotor is now synchronously excitated by an unbalance mass and the stiffness of the bearing are introduced as design variables. The problem of optimization was firstly formulated by keeping the distance from the disk to the left end constant (y_c = 1/3 L), and subsequently, as a design variable.

Max η_i
$15º \leq \varphi_1 \leq 75º$
$15º \leq \varphi_2 \leq 75º$
$1.10^7 \leq K_{xx} \leq 1.10^8$
$5.10^7 \leq K_{zz} \leq 8.10^8$
$0.2 \leq y_c \leq 0.4$

The data of the rotor are the same given in section 5.1.

Figures (7) and (8) show the optimal solution as well as the solution for $\varphi_1 = \varphi_2$ in the range of 15º to 75º, for the rotor in carbon/epoxy and in glass/epoxy. The optimum (a) and optimum (b) refer to optimal solution with and without the design variable y_c respectively. The results are shown on the Tab. (3). The optimal results are according to the theory because, as major is the anisotropie of the bearings, major is the internal damping introduced by the wounding shaft, Silveira (2001).

Table 3. Optimal solution for rotor with synchronous excitation.

Optimal solution					
	φ_1	φ_2	K_{xx} (MPa)	K_{zz} (MPa)	y_c (m)
Carbon/epoxy (y_c constant)	35,72º	34,31º	$1,10^7$	$8,10^8$	(0,333)
carbon/epoxy (y_c variable)	36,27º	36,27º	$1,10^7$	$8,10^8$	0,4
glass/epoxy (y_c constant)	30,76º	30,76º	$1,10^7$	$8,10^8$	(0,333)
glass/epoxy (y_c variáble)	31,53º	31,53º	$1,10^7$	$8,10^8$	0,4

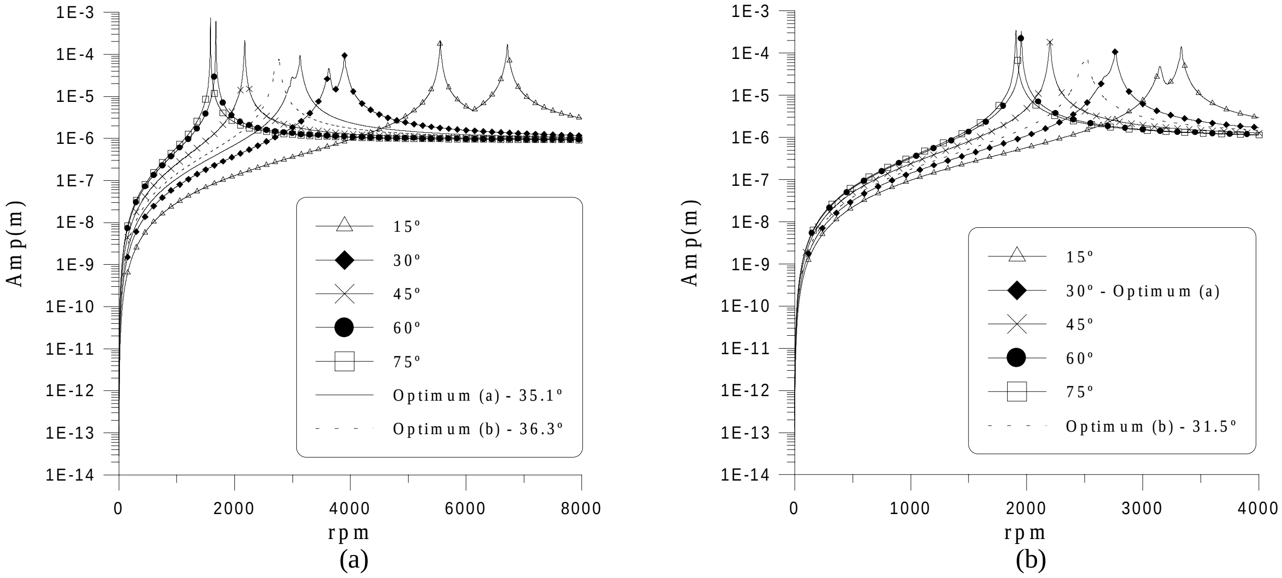


Figure 7. Figure 5. Response to an unbalance mass for rotors in: (a) carbon/epoxy; (b) glass/epoxy.

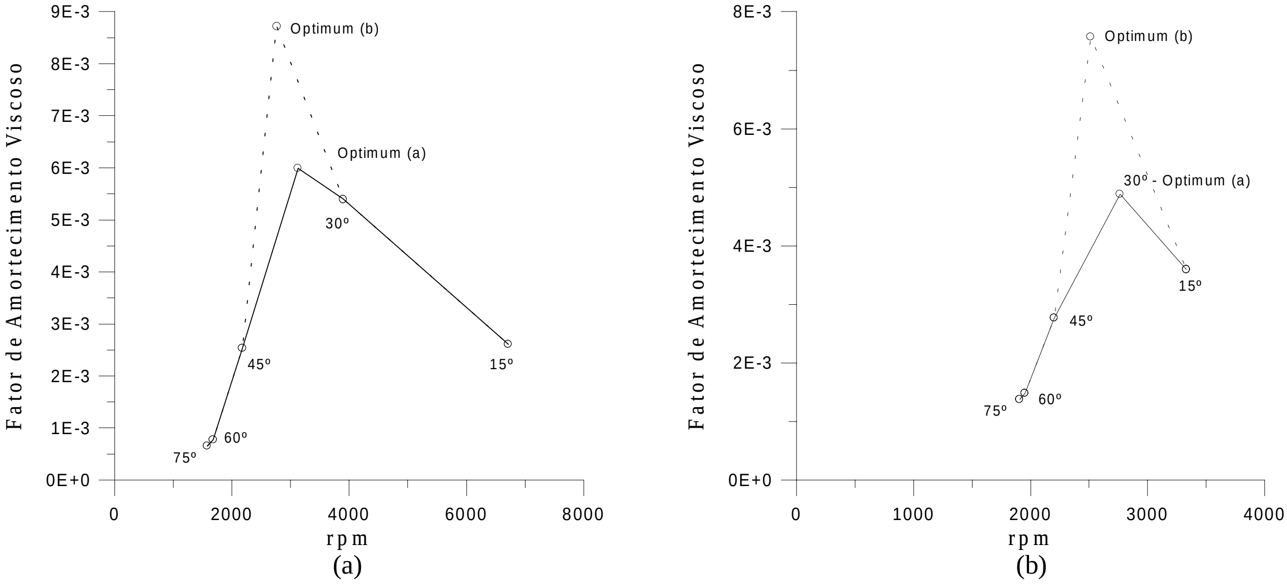


Figure 8. Loss factor on the first crítical speed: (a) carbon/epoxy; (b) glass/epoxy

6. Conclusion

In this work it was investigated the unbalance response on carbon/epoxy and glass/epoxy rotors with isotropic bearings unsynchronously excitated and with anisotropic bearings synchronously excitated. It was observed that composite materials can improve a large number of design variables in rotordynamics analysis and this can be explored in order to search the optimal design. The large difference between the stiffness on the longitudinal and on the tranversal direction of the fiber as for the carbon/epoxy (see Table 1) can be advantageous in order to keep the critical velocity far from the work rotation.

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